Effective Service Capacity of Cellular Systems

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Outline

- Introduction
- Cases of noise-limited systems
- Cases of interference-limited systems
- Numerical results
- Conclusions

Motivation

Much effort invested to enhance cellular spectral efficiency:

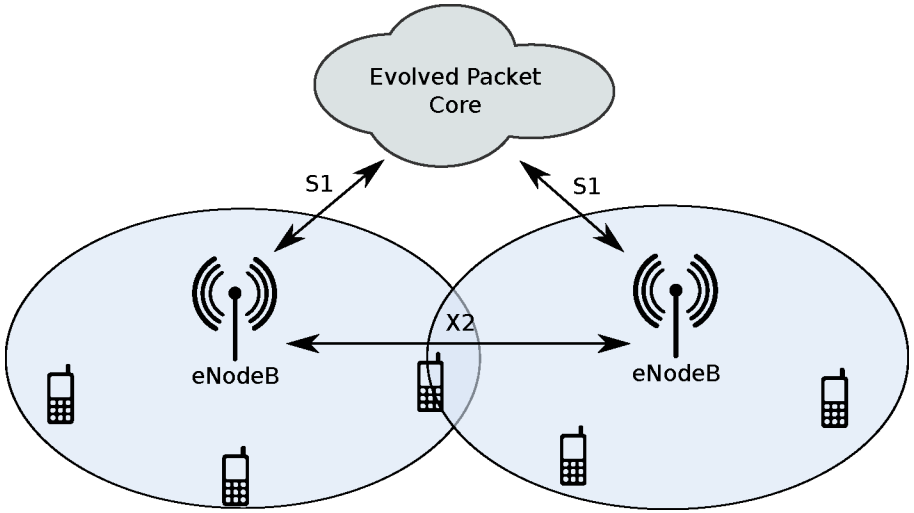
- Multi-carrier (OFDM, SC-FDMA)
- Multi-antenna (Beamforming, CoMP)
- Dynamic spectrum allocation
- Interference coordination / alignment
- Relaying

Focus on physical layer performance

Metrics: Shannon capacity, outage-limited capacity etc.

Motivation II

More practical questions arise from the usage in real systems



➔ Quantize the queuing performance of different approaches

Analysis Approaches

Older approaches for evaluating queuing performance:

- Utilize the M/G/1 framework
- Matrix methods

But: Arrivals not necessarily Markovian, no delay distribution

Turn to newer approaches:

- Effective capacity
- Stochastic network calculus

Effective Capacity

Introduced by [Wu, ToW, 2003]:

- Q denotes random steady-state queue length
- From large deviation theory we get approximation

$$\Pr.\{Q > x\} \approx K \cdot e^{-\theta \cdot x} \quad \Rightarrow \quad \Pr.\{D > d\} = \mathcal{P} \approx K \cdot e^{-\theta \cdot r \cdot d}$$

- K is the probability that the queue is not empty
- θ is the QoS exponent with $r \leq \alpha(\theta) = -\frac{\Lambda(-\theta)}{\theta}$

with $\Lambda(\theta) = \lim_{i \to \infty} \frac{1}{i} \log \mathrm{E}\left[e^{\theta \cdot (S[i] - S[0])}\right]$ as the log-moment generating function and $\alpha(\theta)$ as the effective service capacity

Effective Capacity

Introduced by [Wu, ToW, 2003]:

$$r_j \leq \alpha(\theta) = -\frac{\Lambda(-\theta)}{\theta}$$

Extended by [Soret, CeDG 2010]:

- With central limit theorem (if service process increments are i.i.d)

$$\alpha(\theta) = \mathrm{E}\left[s_j[i]\right] - \frac{\theta}{2}\mathrm{Var}\left[s_j[i]\right]$$

- Max. sustainable rate:

$$r_j^* \approx \frac{1}{2} \cdot \left(\mathrm{E}\left[s_j\right] + \sqrt{(\mathrm{E}[s_j])^2 + \frac{2 \cdot \ln(\mathcal{P}_j)}{d_j} \cdot \mathrm{Var}[s_j]}\right)$$

Our Efforts

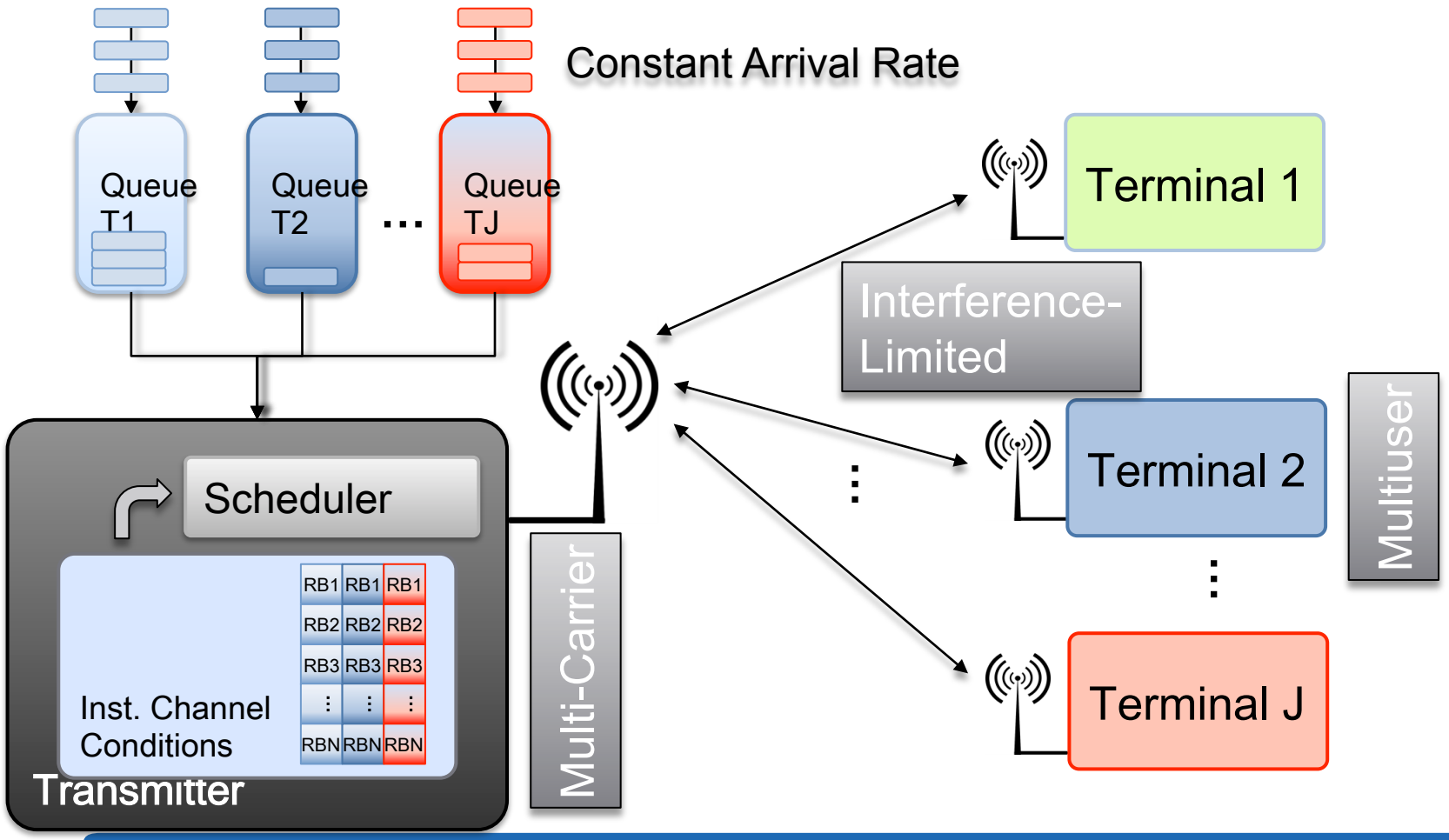
Analyze common OFDM schedulers in:

- Noise-limited environments
- Interference-limited environments
- Dynamic spectral reuse scenarios

➔ Determine the impact on the queuing performance

➔ Utilize results for operation of cellular networks

System Model



Noise Limited Round Robin Scheduler

$$f_{\bar{\gamma}_j}^{\mathrm{RRN}}(x) = \frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}} \cdot e^{-\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}} x}, \quad F_{\bar{\gamma}_j}^{\mathrm{RRN}}(x) = 1 - e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}} x}$$

$$\begin{aligned} \mathrm{E}^{\mathrm{RRN}}\left[s_j\right] &= \mathcal{S} \cdot \sum_{n=1}^{N} \sum_{i=0}^{M-1} c_i \int_{\gamma_i}^{\gamma_{i+1}} \frac{f_{\bar{\gamma}_{j,n}}^{\mathrm{RRN}}(x)}{J} dx \\ &= \frac{N}{J} \cdot \mathcal{S} \cdot \sum_{i=0}^{M-1} c_i \cdot \left[1 - e^{-\frac{\sigma^2 x}{\mathrm{P}_{\mathrm{S}}}}\right]_{\gamma_i}^{\gamma_{i+1}} \end{aligned} \tag{1}$$

Noise Limited opp. OFDMA Scheduler

$$\approx \frac{1}{2}\left(\int_0^{\gamma_{m+1}} f_{\bar{\gamma}_{l,n}}(y)dy + \int_0^{\gamma_m} f_{\bar{\gamma}_{l,n}}(y)dy\right)$$

$$\mathrm{E}^{\mathrm{OON}}\left[s_j\right] = \sum_{n=1}^{N} \mathrm{E}\left[s_{(J/J),n}\right]$$

$$= \sum_{n=1}^{N}\sum_{m=0}^{M-1}\int_{\gamma_m}^{\gamma_{m+1}} \mathcal{S}\cdot c_m \cdot f_{\bar{\gamma}_{j,n}}(x)\cdot \prod_{\substack{l=1\\ j\neq l}}^{J}\int_0^x f_{\bar{\gamma}_{l,n}}(y)dydx$$

$$= N\cdot\sum_{m=0}^{M-1}\int_{\gamma_m}^{\gamma_{m+1}} \mathcal{S}\cdot c_m \cdot f_{\bar{\gamma}_{j,n}}(x)\cdot \prod_{\substack{l=1\\ j\neq l}}^{J} F_{\bar{\gamma}_{l,n}}(x)dx$$

$$= N\cdot\sum_{m=0}^{M-1}\int_{\gamma_m}^{\gamma_{m+1}} \mathcal{S}\cdot c_m \cdot \frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}}\cdot e^{-\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}}x}\cdot \prod_{\substack{l=1\\ j\neq l}}^{J}\left(1-e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},l}}x}\right)dx$$

2^J parts

Noise Limited opp. OFDMA scheduler

$$
\begin{aligned}
\mathrm{E}^{\mathrm{OON}}\left[s_j\right] &= \sum_{n=1}^{N} \mathrm{E}\left[s_{(J/J),n}\right] \\
&= \sum_{n=1}^{N} \sum_{m=0}^{M-1} \int_{\gamma_m}^{\gamma_{m+1}} \mathcal{S} \cdot c_m \cdot f_{\bar{\gamma}_{j,n}}(x) \cdot \prod_{\substack{l=1 \\ j \neq l}}^{J} \int_{0}^{x} f_{\bar{\gamma}_{l,n}}(y) dy dx \\
&\approx N \cdot \sum_{m=0}^{M-1} \int_{\gamma_m}^{\gamma_{m+1}} \mathcal{S} \cdot c_m \cdot f_{\bar{\gamma}_{j,n}}(x) \cdot \prod_{\substack{l=1 \\ j \neq l}}^{J} \frac{1}{2} \left(F_{\bar{\gamma}_{l,n}}(\gamma_{m+1}) + F_{\bar{\gamma}_{l,n}}(\gamma_m) \right) dx \\
&= N \cdot \sum_{m=0}^{M-1} \prod_{\substack{l=1 \\ j \neq l}}^{J} \frac{1}{2} \left(\left(2 - e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},l}} \gamma_{m+1}} - e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},l}} \gamma_m} \right) \right) \cdot \mathcal{S} \cdot c_m \cdot \left[1 - e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}} x} \right]_{\gamma_m}^{\gamma_{m+1}}
\end{aligned}
$$

Interference Limited Systems

Modelling possibilities:

- Treat interference as additional noise
  - Different noise levels for each terminal?
  - Worst case analysis with worst interference?

- Use distribution function for interference limited signals

$$f(x) = \left[\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}x+\mathrm{P}_{\mathrm{S},j}} + \frac{\mathrm{P}_{\mathrm{I},j}\mathrm{P}_{\mathrm{S},j}}{(\mathrm{P}_{\mathrm{I},j}x+\mathrm{P}_{\mathrm{S},j})^2}\right] \cdot e^{-\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}}x},$$

$$F(x) = 1 - \frac{\mathrm{P}_{\mathrm{S},j}}{\mathrm{P}_{\mathrm{I},j}x+\mathrm{P}_{\mathrm{S},j}} \cdot e^{-\frac{\sigma^2}{\mathrm{P}_{\mathrm{S},j}}x}$$

$\Rightarrow$ Subcarriers independent but non identically distributed

Interference limited prop. fair scheduler

Normalize instantaneous SINR with average SINR

$$\mathrm{E}^{\mathrm{I}}\left[\gamma_j\right] = \frac{\mathrm{P}_{\mathrm{S},j}}{\mathrm{P}_{\mathrm{I},j}} \cdot e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)$$

$\Rightarrow$ Distribution functions are now:

$$f_{\bar{\gamma}_j}^{\mathrm{PFN}}(x) = e^{-x} \qquad \Rightarrow f_{\bar{\gamma}_j}^{\mathrm{PFI}}(x) = \left(\frac{e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)}{(e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)x+1)^2} + \frac{\sigma^2 e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)}{\mathrm{P}_{\mathrm{I},j}(e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)x+1)}\right) \cdot e^{-\frac{\sigma^2 e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)}{\mathrm{P}_{\mathrm{I},j}}x}$$

$$F_{\bar{\gamma}_j}^{\mathrm{PFN}}(x) = 1 - e^{-x} \qquad \Rightarrow F_{\bar{\gamma}_j}^{\mathrm{PFI}}(x) = 1 - \frac{1}{e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)x+1} \cdot e^{-\frac{\sigma^2 e^{\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}} \cdot \mathrm{E}_1\left(\frac{\sigma^2}{\mathrm{P}_{\mathrm{I},j}}\right)}{\mathrm{P}_{\mathrm{I},j}}x}$$

$\Rightarrow$ No longer identically distributed

Numerical Results – Interference limited Systems

How to handle interference limited systems?

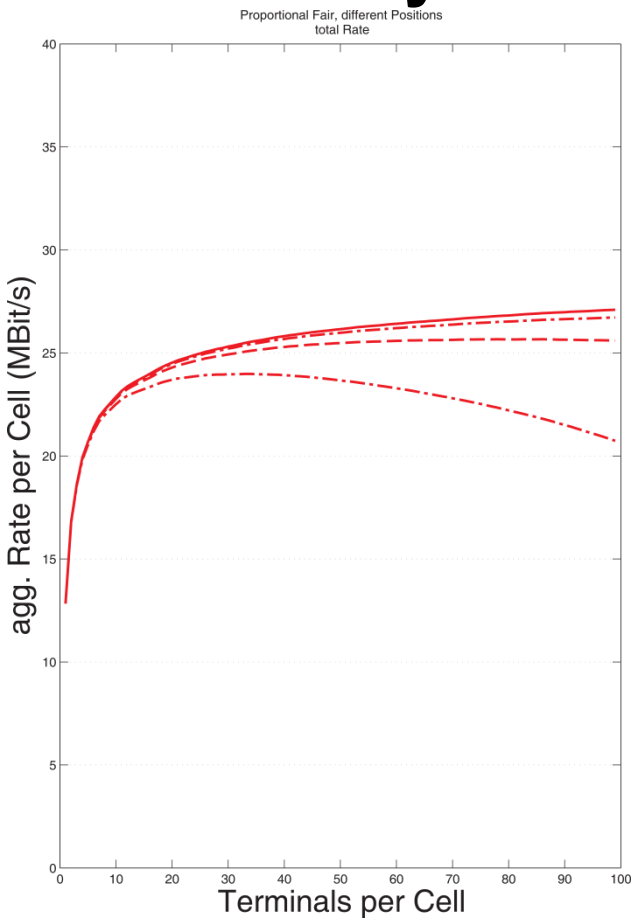
- Interference as noise ?

Rates for Terminals with different Noise levels

SNR between (5.68dB , 6.10dB)

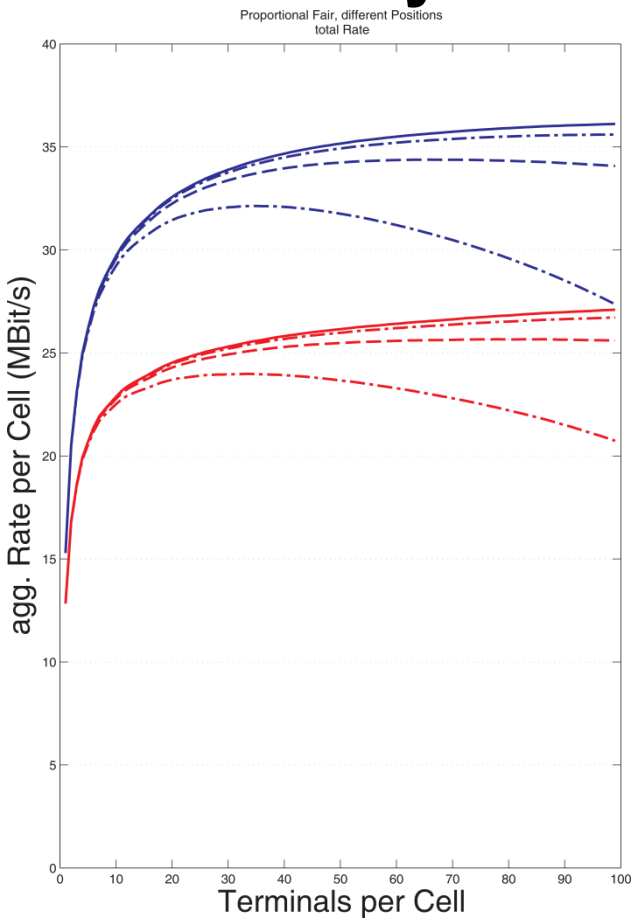
SINR between (5.68dB, 6.10dB)

Interference limited Systems – prop. Fair



Int as Noise

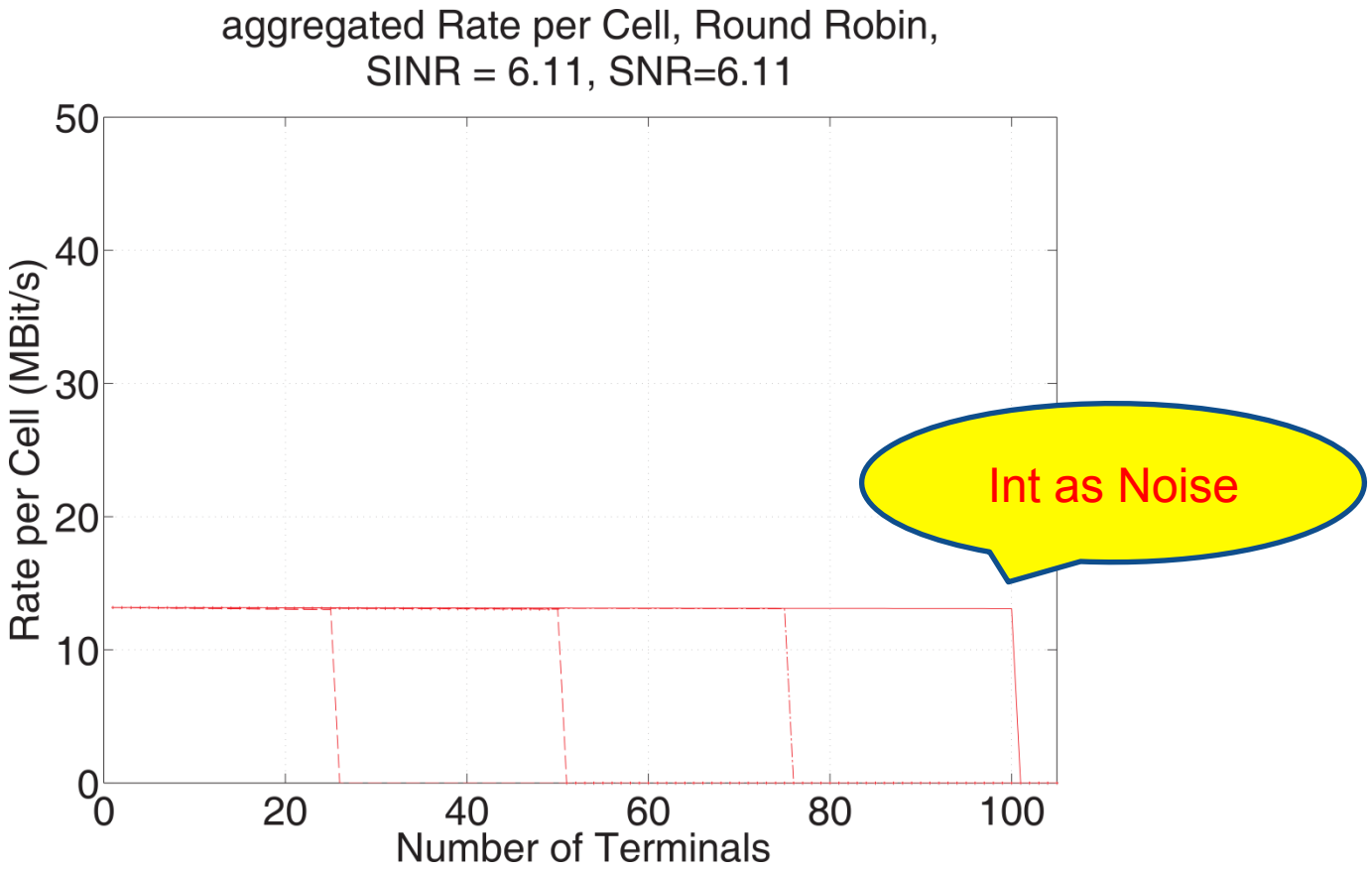
Interference limited Systems – prop. Fair



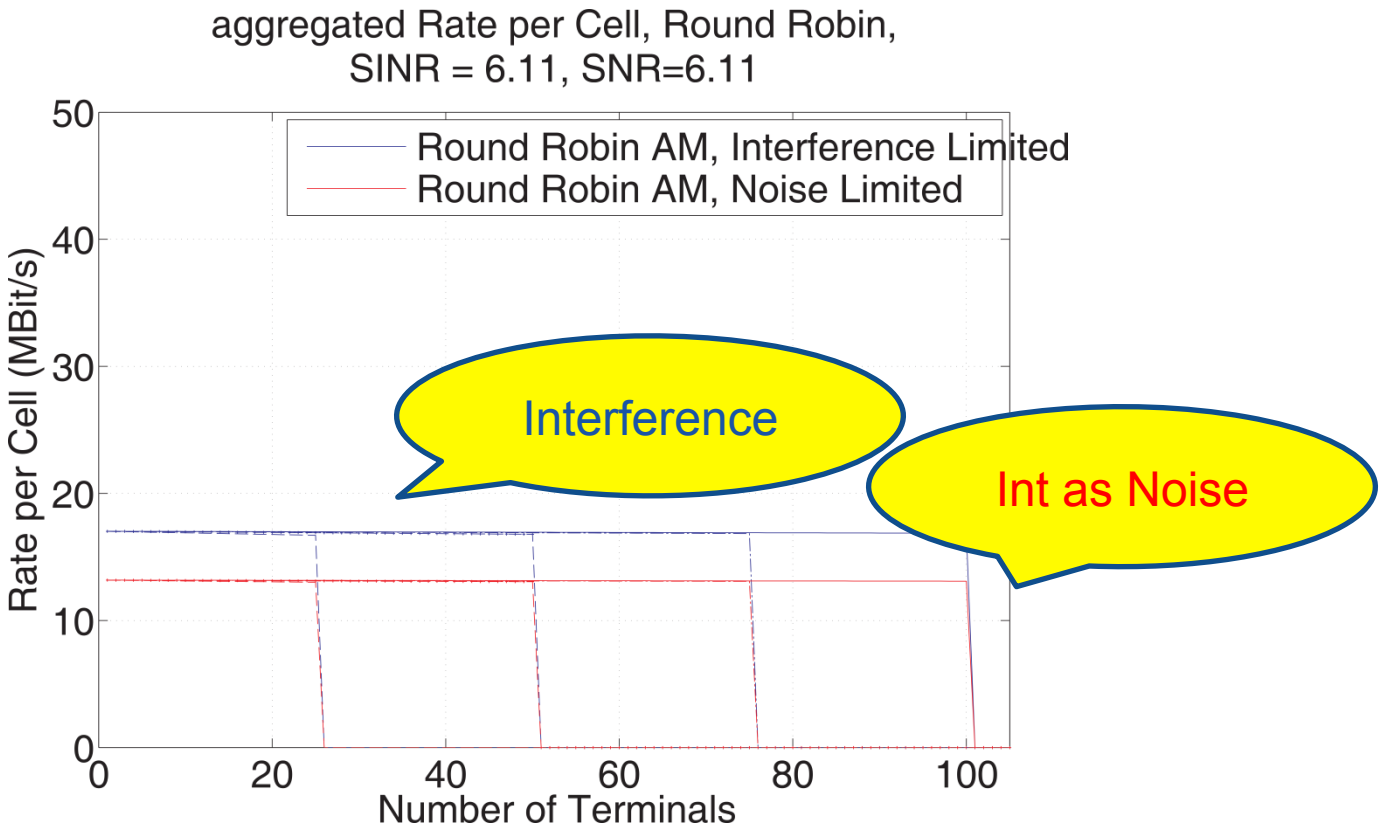
Interference

Int as Noise

Interference limited Systems



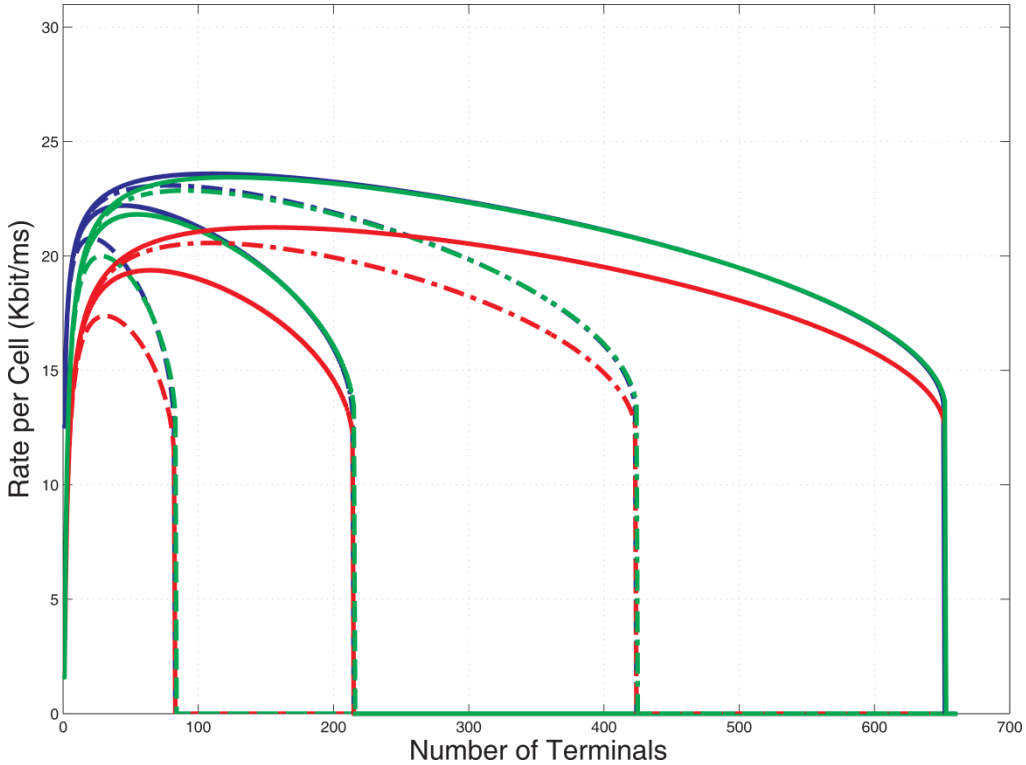
Interference limited Systems



Conclusion

- Effective Capacity good tool for analyzing cellular wireless systems
  - Intuitive parameters like delay in ms or outage probability
- Works for realistic multiuser multichannel systems
- Modelling interference as additional noise leads to an underestimation of the system

Numerical Results



Opp. OFDMA
AM
LA
LApT

Notation

$$
\begin{aligned}
\mathrm{P}_{\mathrm{S},j} &= \text{Avg. transmitter gain for MS } j \\
\mathrm{P}_{\mathrm{I},j} &= \text{Avg. interferer gain for MS } j \\
\sigma^2 &= \text{Noise} \\
\bar{\gamma}_{j,n} &= \text{Avg. SNR of MS } j \text{ on SC } n \\
\mathcal{S} &= \text{Symbols per TTI} \\
N &= \text{Number of SC} \\
J &= \text{Number of MS} \\
M &= \text{discrete capacity function with } M \text{ steps} \\
c_i &= \text{capacity for SNR in intervall } [\gamma_i, \gamma_{i+1}),\ i \in [0, \ldots, M-1]
\end{aligned}
$$