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A Network Calculus Modeling Flow Transformations with Variable Packet Lengths

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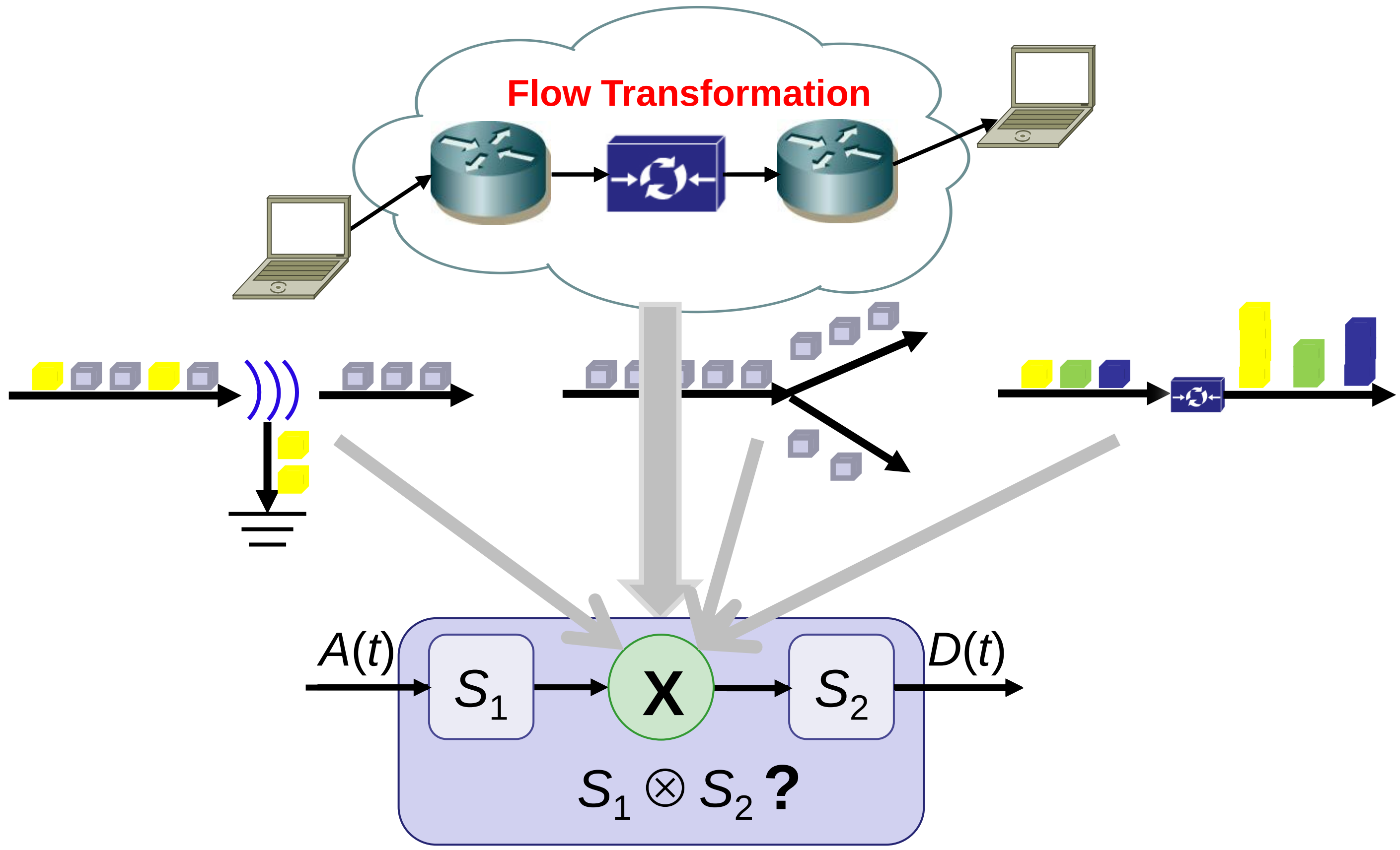
Recall Model of Flow Transformation



Scaling element : no buffer,

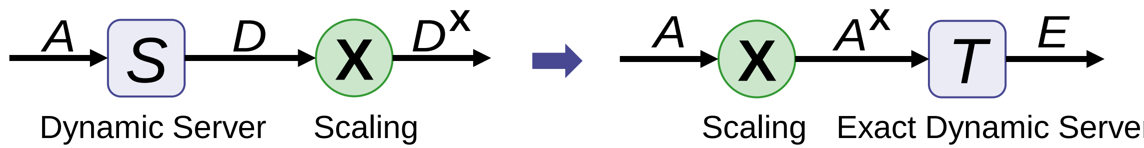
$\mathbf{X} = (X_i)_{i \geq 1}$ non-negative integers,

scaled process $A^{\mathbf{X}}(t) = \sum_{i=1}^{A(t)} X_i$

Does Convolution-Form still apply to the end-to-end analysis?

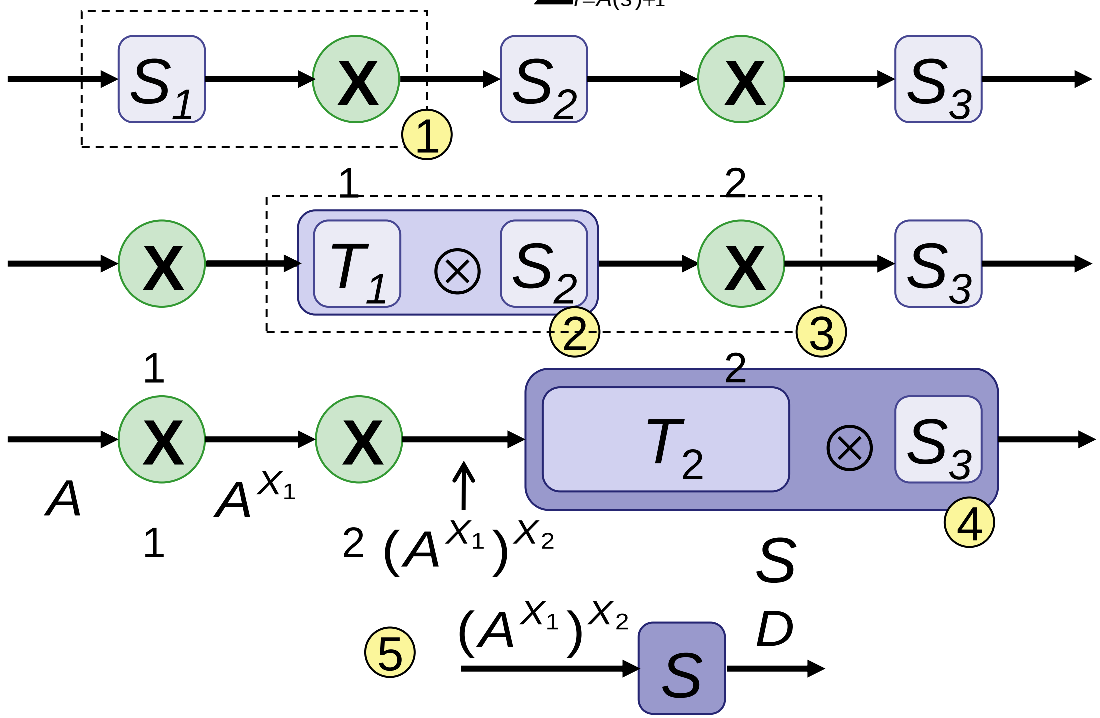
End-to-End Analysis

- To preserve convolution-form: **commutation**



(a) (b)

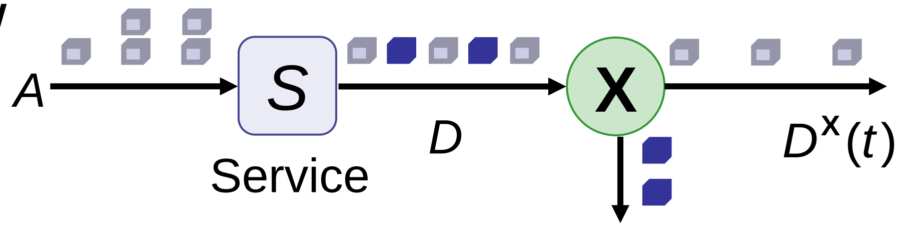
Define an exact server $\boldsymbol{T(s,t)} := \sum_{i=A(s)+1}^{A(s)+S(s,t)} X_i$, such that $E(t) \leq D^X(t)$.



"On Expressing Networks with Flow Transformations in Convolution-Form," by Florin Ciucu, Jens Schmitt, Hao Wang, INFOCOM 2011

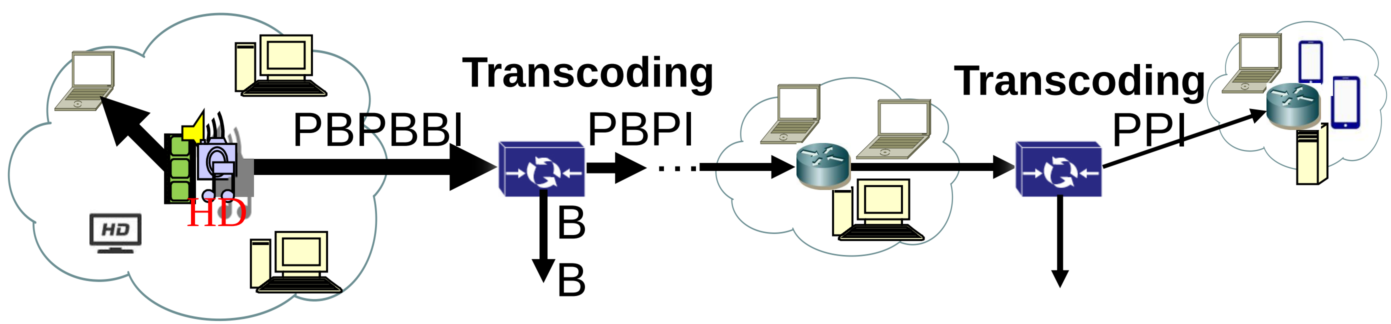
A Case of Flow Transformation - Loss

- Review



$$D^{X}(t) = \sum_{i=1}^{D(t)} X_i = \sum_{i=1}^{D(t)} 1 \cdot X_i$$

- However, sometimes the flows are of packets with variable lengths and hard to know the scaling information of each bit

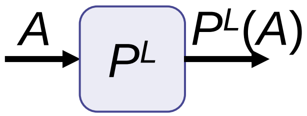


More realistic to know the scaling information of each packet!

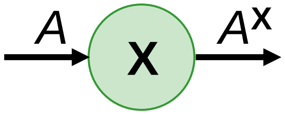
➡Use packetizer + define packet scaling element

Packet Scaling Element

Packet process L: $L(n) = l_0 + l_1 + \cdots + l_n, n = 0, 1, 2, \cdots$
Packetized process: $P^L\left(A(t)\right) = \sum_{i=1}^{N_t} l_i$, where
$N_t = \max\{m : \sum_{i=1}^{m} l_i \leq A(t)\}$. Denote **packetizer** as P^L.

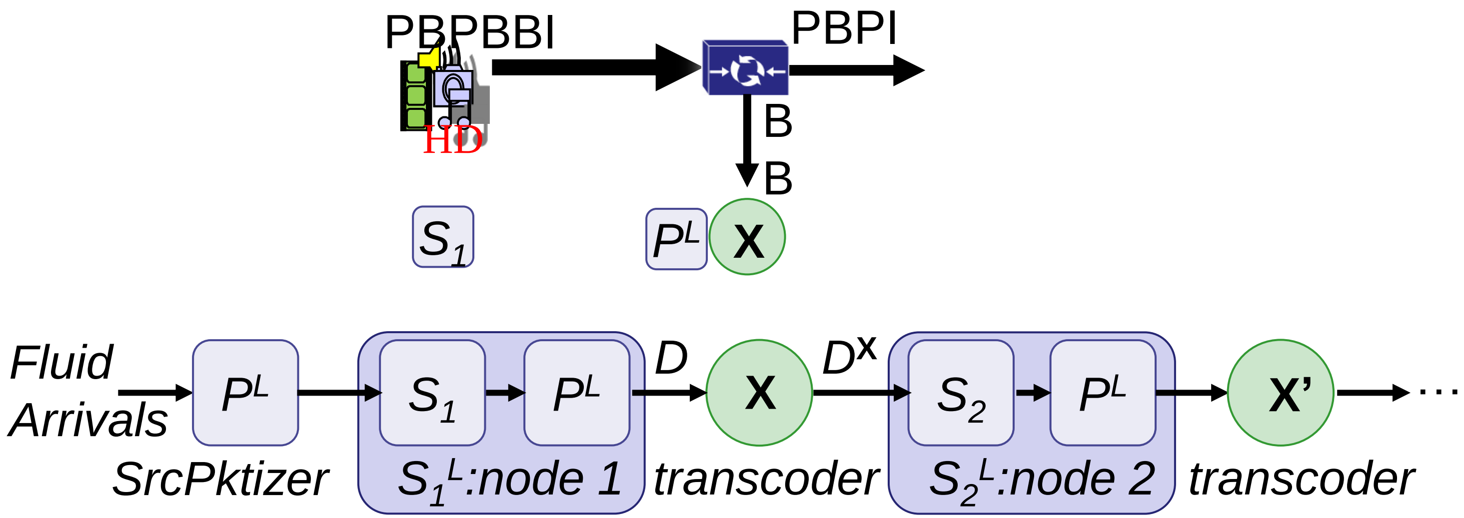


A **packet scaling element** consists of an L-packetized arrival process $A(t) = \sum_{i=1}^{N_t} l_i$, a packet scaling process $\mathbf{X}$ taking non-negative integer values and a scaled packetized flow defined for all $t \geq 0$ as $A^{\mathrm{X}}(t) = \sum_{i=1}^{N_t} \mathbf{l}_{\mathrm{i}} X_i$.



$W(t)$ is called **packet delay**, if for all $t \geq 0$
$W(t) = \inf\left\{d \geq 0 : P^L(A(t)) \leq P^L(D(t+d))\right\}$.

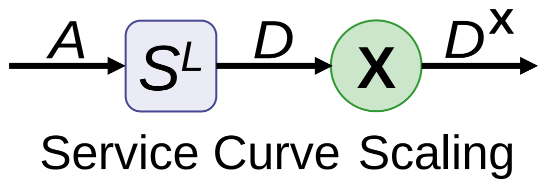
Model with Packetizer and Packet Scaling

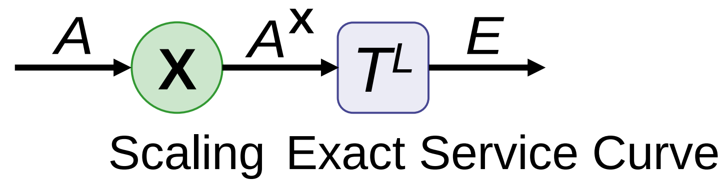


- Challenges
  - Commutation to reserve convolution-form
  - Dynamic server of service with packetizer S^L
  - MGF bound of scaled process
  - Capture randomness from packet process

An Approach

- Commutation



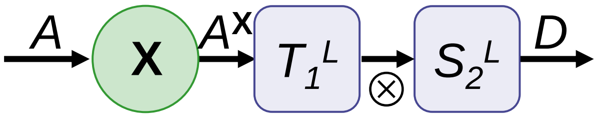


We define $T^L(s,t) := \sum_{i=M_s+1}^{N_{s,t}} l_i X_i$ as the exact service curve, where $A(s) = \sum_{i=1}^{M_s} l_i$, $A(s) + S^L(s,t) = \sum_{i=1}^{N_{s,t}} l_i$.

- A packetized server S^L consists of a dynamic server S and a packetizer P^L.

If the maximal packet length l_{max} exists, a possible dynamic server is $S^L(s,t) = [S(s,t) - l_{max}]_+$.

End-to-end Delay: Two Nodes Case (1)



$$
\begin{aligned}
& Pr(W(t) > d) \\
= \quad & Pr\left(P^L\left(A^X(t)\right) > P^L\left(D(t+d)\right)\right) \\
\leq \quad & Pr\left(A^X(t) > A^X \otimes \left(T_1^L \otimes S_2^L\right)(t+d)\right) \\
= \quad & Pr\left(\sup_{0 \leq s \leq t+d} \left\{A^X(t) - A^X(s) - T_1^L \otimes S_2^L(t+d) > 0\right\}\right) \\
\leq \quad & \sum_{0 \leq s < t} \sum_{s \leq u \leq t+d} Pr\Big(A^X(t) - A^X(s) - T_1^L(s,u) - S_2^L(u,t+d) > 0\Big) \\
= \quad & \sum_{0 \leq s < t} \sum_{s \leq u \leq t+d} Pr\left(\sum_{i=1}^{N_t} l_i X_i - \sum_{i=1}^{N_s} l_i X_i - \sum_{i=N_s+1}^{N_{s,u}} l_i X_i > S_2^L(u,t+d)\right) \\
& \text{where } \sum_{i=1}^{N_t} l_i = A(t), \sum_{i=1}^{N_s} l_i = A(s), \sum_{i=1}^{N_{s,u}} l_i = A(s) + S_1^L(s,u) \\
= \quad & \sum_{0 \leq s < t} \sum_{s \leq u \leq t+d} Pr\left(\sum_{i=1}^{N_t} l_i X_i - \sum_{i=1}^{N_{s,u}} l_i X_i > S_2^L(u,t+d)\right) \\
& \ldots
\end{aligned}
$$

End-to-end Delay: Two Nodes Case (2)

Assume that $\mathbf{X}$ is a stationary Markov process and l_i's are *i.i.d.* ,
$l_i X_i, i \geq 1$ is a Markov modulated process (MMP) .

$$Pr(W(t) > d)$$

$$\leq \sum_{0 \leq s < t} \sum_{s \leq u \leq t+d} Pr\left(\sum_{i=1}^{N_t} l_i X_i - \sum_{i=1}^{N_{s,u}} l_i X_i > S_2^L(u, t+d) \right) \quad \text{stationary}$$

$$= \sum_{0 \leq s < t} \sum_{s \leq u \leq t+d} Pr\left(\sum_{i=1}^{N_t - N_{s,u}} l_i X_i > S_2^L(u, t+d) \right) \quad \text{dependent or independent}$$

Assume $S_i^L(s,t) = [S_i(s,t) - l_{max}]_+$

$$\leq \sum_{0 \leq s < t} \sum_{s \leq u \leq t+d} E\left[e^{-\theta S_2^L(u,t+d)} \right] E\left[e^{\theta \sum_{i=1}^{N_t - N_{s,u}} l_i X_i} \right]$$

MMP

$$\leq \sum_{0 \leq s < t} \sum_{s \leq u \leq t+d} E\left[e^{-\theta S_2^L(u,t+d)} \right] E\left[e^{\theta R(\theta)(N_t - N_{s,u})} \right] \quad \theta\text{-MER}$$

$$M_{A(t)-A(s)-S_1^L(s,u)}(\theta') = E\left[e^{\theta' \sum_{i=1}^{N_t - N_{s,u}} l_i} \right] \qquad E\left[e^{\theta'(A(t)-A(s))} e^{-\theta' S_1^L(s,u)} \right]$$

$$= E\left[e^{\log M_{l_1}(\theta')(N_t - N_{s,u})} \right] \qquad \leq e^{\theta' r(\theta')(t-s)} e^{-\theta' S_1^L(s,u)}$$

Let $\theta R(\theta) = \log M_{l_1}(\theta')$ and $b = \sup \{ e^{-\theta C_2}, e^{-\theta' C_1} \}$

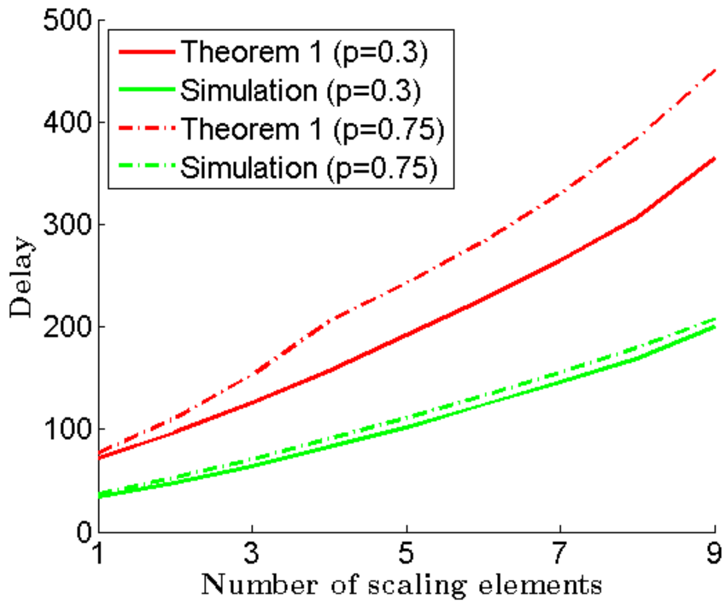
$$\leq e^{-\theta C_2 d} e^{(\theta + \theta') l_{max}} \sum_{s \leq u \leq t+d} 1 \sum_{0 \leq s < t} e^{(\log b + \theta' r(\theta'))(t-s)}$$

End-to-end Delay: N Nodes Case

L-packetized arrivals, stationary and (mutually) independent bit level service $S_1, S_2, \cdots, S_n$, scaling elements are *i.i.d.* loss processes $X_1, X_2, \cdots, X_{n-1}$. Assume l_i's are *i.i.d.* . $M_{A(s,t)}(\theta) \leq e^{\theta r_A(\theta)(t-s)}$ and $M_{S_k(s,t)}(-\theta) \leq e^{-\theta C_k(t-s)}$.

$$Pr(W > d) \leq e^{\left(\sum_{i=1}^{n} \theta_i + \theta_1\right) l_{max}} K^n b^d ,$$

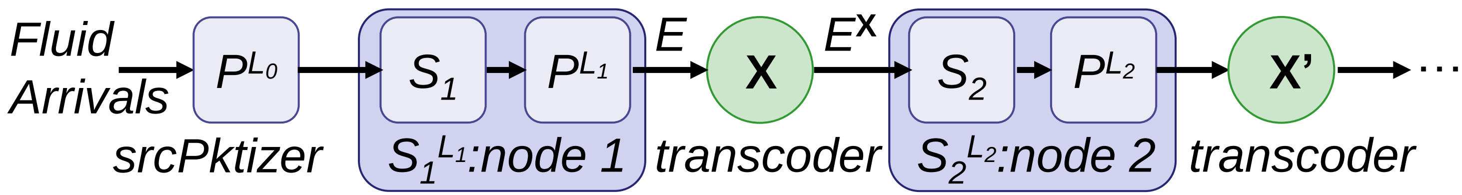
where $\theta_n R_{n-1}(\theta_n) = \log M_l(\theta_{n-1})$ and $b = \sup e^{-\theta_k C_k} : 1 \leq k \leq n$.



Poisson bits arrival $Poi(1)$ packetized by *i.i.d.* l_i's $\sim U[1, 16]$; $\mathbf{X}_i \sim B(p_i)$ and let $p_i = p$; $C_1, \cdots, C_{10} = 1.25, 1.15, 1.05, 0.95, 0.85, 0.80, 0.75, 0.70, 0.65, 0.60$ $\varepsilon = 10^{-3}$

Some Thoughts

- Heterogeneous packetizers
  - Bit delay, dynamic packetized server, packet scaling to bit scaling



- Consider the packet distribution in flow transformation
  - Tail bound adapted for analysis with packet distribution but not for scaling element

    ➡ Capture the scaling information in form of tail bound instead of MGF bound
- Without packetizers, still packet scaling
- Packet scaling information harder, then bit scaling

Conclusion

- Model flow transformation with “bit”-level scaling element

- Model flow transformation with variable packet lengths
  - Motivation
  - Definition of packet scaling element
  - Commutation

- End-to-end delay analysis under simple assumptions

- Some thoughts

Thank you!
Questions, comments, …