Service Characterizations and Performance Analysis for Multi-Hop Multiaccess Wireless Channels

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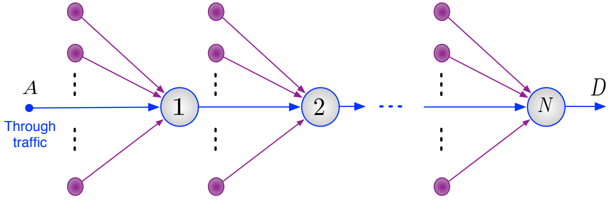
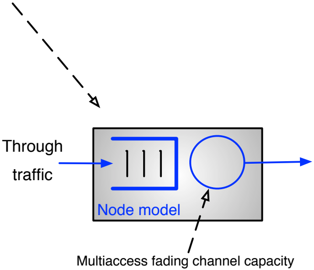
2nd Workshop on Network Calculus (WoNeCa-2)
Bamberg, Germany, March 2014

Multihop Multiaccess Wireless Network Model



- Intermediate nodes are store and forward relays
- Channel capacity at each node is shared by L homogeneous flows
- Service offered to A is given by the multiaccess fading channel capacity



How to model multiaccess fading channel capacity?

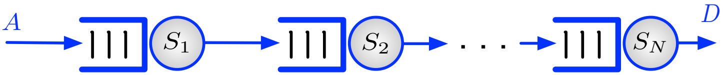
Multiaccess Wireless Channels

Multiple transmitters sharing a single channel. Three approaches:

- Multiaccess information theory [Ahlswede 1971, Liao 1972]
  - physical layer approach
  - coding schemes for reliable many–to–one communication
  - concurrent transmissions with no coordination
  - ignores burstiness, i.e., permanently backlogged flows
- Random access or collision resolution [Abramson 1970]
  - operates in the MAC layer
  - no transmissions coordination (collision is possible)
  - no more than one user can transmit (successfully) at a time
  - considers burstiness and interference but not noise and fading
- Dynamic scheduling
  - network layer approach
  - channel is viewed as a bit pipe with randomly varying service
  - scheduler is channel aware and opportunistic
  - centralized decision: one user transmits at a time

$\Rightarrow$ **different interference types lead to different service models**

Network Model



- Fluid-flow traffic, discrete time
- Arrival and service are independent
- Interfering flows are i.i.d. and their channels have i.i.d. gains
- Time-varying random service that is equal to the Instantaneous multiaccess channel capacity
- Service characterization for each multiaccess approach
- We ignore control overhead

Q: How does the use of different multiaccess approaches impact the end–to–end network layer performance for the through flow?

Related Work

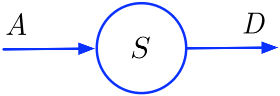
- Different approaches for multiaccess communication were identified by [Gallager 1984]
  - distinct communities $\Rightarrow$ different models of the same problem
  - the goal was to contrast and compare these approaches
- Information–theoretic approach to random access
  - a channel coding that considers burstiness and packet collision detection is proposed by [Luo and Ephremides 2012]
  - information–theoretic formulation is presented and the set of achievable rates is characterized in [Minero et al. 2012]
- Network calculus approach for information–theoretic models analysis
  - Network calculus approach to information theoretic models [Ciucu 2011] and [Lubben and Fidler 2012]
  - A $(\min, \times)$ network calculus for wireless fading channels performance analysis [Al-Zubaidy et al. 2013]

Suggest network calculus for multiaccess networks analysis!

Our Approach

- **Goal:** Compare and reconcile multiaccess solutions emerging from the three different areas
- **Challenges:** Three multiaccess approaches
  - by different communities whom have different perspectives
  - three different models with incongruent assumptions
- **Analysis approach:** Network calculus
- **Main idea:**
  - obtain service characterizations for the three channel models
  - apply network calculus to obtain end–to–end performance bounds using each of the three service characterizations
  - compare bounds

Network Calculus

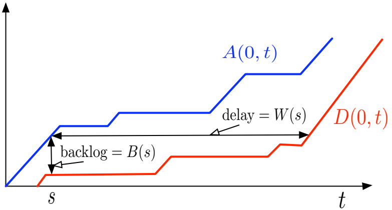


- $(\min, +)$ dioid algebra
- Backlog: $B(s) = A(0,s) - D(0,s)$
- Delay: $W(s) = \inf\{u \geq 0 : A(0,s) \leq D(0, s+u)\}$
- Dynamic server [Chang 2000]

$$D(0,t) \geq \inf_{u \leq t}\{A(0,u) + S(u,t)\}$$
$$= A * S(0,t)$$

- Network service (multi node):

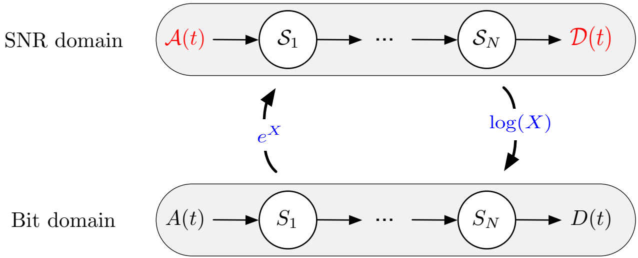
$$S_{\text{net}}(\tau, t) = S_1 * S_2 * \cdots * S_N(\tau, t)$$



- Traffic and service measured in bits, hence, we refer to this analysis domain as 'Bit domain'

Network Calculus for Wireless Networks Analysis

- Analysis of multihop wireless networks in the bit domain is difficult!
- Instead we conduct our analysis in an alternative domain (SNR domain) that was suggested in [Al-Zubaidy et al. 2013]



- Transfer between the two domains using logarithmic functions

Service and Traffic Elements in the SNR Domain

- Service process for fading channel in the bit domain:

$$S(\tau, t) = \sum_{i=\tau}^{t-1} \log g(\gamma_i)$$

The function $g(\gamma_i)$ encodes the effect of the sharing mechanism for each of the three multiaccess channels

- Service process in the SNR domain

$$\mathcal{S}(\tau, t) = e^{S(\tau,t)} = \prod_{i=\tau}^{t-1} g(\gamma_i) \qquad \Longrightarrow \text{ simpler}$$

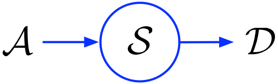
- SNR traffic processes:

$$\mathcal{A}(\tau, t) = e^{A(\tau,t)} \quad \text{and} \quad \mathcal{D}(\tau, t) = e^{D(\tau,t)}$$

$\Longrightarrow$ SNR domain is governed by $(\min, \times)$ dioid algebra

$(\min, \times)$ Network Calculus

- Service: $\mathcal{S}(\tau, t) = \prod_{i=\tau}^{t-1} g(\gamma_i)$
- Arrival: $\mathcal{A}(\tau, t) = \prod_{i=\tau}^{t-1} e^{a_i}$



$(\min, \times)$-convolution: $\mathcal{X} \otimes \mathcal{Y}(\tau, t) \stackrel{\triangle}{=} \inf_{\tau \le u \le t} \{\mathcal{X}(\tau, u) \cdot \mathcal{Y}(u, t)\}$

$(\min, \times)$-deconvolution: $\mathcal{X} \oslash \mathcal{Y}(\tau, t) \stackrel{\triangle}{=} \sup_{u \le \tau} \left\{ \frac{\mathcal{X}(u, t)}{\mathcal{Y}(u, \tau)} \right\}$

- Departures: $\mathcal{D}(0, t) \ge \mathcal{A} \otimes \mathcal{S}(0, t)$
- Backlog Bound: $\mathcal{B}(t) = \mathcal{A}(0, t)/\mathcal{D}(0, t) \le \mathcal{A} \oslash \mathcal{S}(t, t)$
- Delay Bound: $\mathcal{W}(t) \le \inf \left\{ d \ge 0 : \mathcal{A} \oslash \mathcal{S}(t + d, t) \le 1 \right\}$
- Network SNR server: $\mathcal{S}_{\text{net}}(\tau, t) = \mathcal{S}_1 \otimes \mathcal{S}_2 \otimes \cdots \otimes \mathcal{S}_N(\tau, t)$

Computation of $\mathcal{S}_1 \otimes \mathcal{S}_2$ and $\mathcal{A} \oslash \mathcal{S}$

- **Mellin transform:** $\mathcal{M}_X(s) = E[X^{s-1}]$
- For two independent processes

$$\mathcal{M}_{\mathcal{X}\otimes\mathcal{Y}}(s,\tau,t) \le \sum_{u=\tau}^{t} \mathcal{M}_{\mathcal{X}}(s,\tau,u) \cdot \mathcal{M}_{\mathcal{Y}}(s,u,t)$$

$$\mathcal{M}_{\mathcal{X}\oslash\mathcal{Y}}(s,\tau,t) \le \sum_{u=0}^{\tau} \mathcal{M}_{\mathcal{X}}(s,u,t) \cdot \mathcal{M}_{\mathcal{Y}}(2-s,u,\tau)$$

- For N i.i.d. fading channels we compute

$$\mathcal{M}_{\mathcal{S}_{\text{net}}}(s,\tau,t) \le \binom{N-1+t-\tau}{t-\tau} \cdot \left(\mathcal{M}_{g(\gamma)}(s)\right)^{t-\tau}, \quad \forall s < 1$$

- **Moment bound**: $Pr(X \ge a) \le a^{-s}\mathcal{M}_X(1+s), \ \forall a, s > 0$

Statistical Performance Bounds

Define

$$\mathsf{M}_{\mathrm{net}}(s,\tau,t) \triangleq \sum_{u=0}^{\min(\tau,t)} \mathcal{M}_{\mathcal{A}}(1+s,u,t)\cdot\mathcal{M}_{\mathcal{S}_{\mathrm{net}}}(1-s,u,\tau)$$

- BACKLOG: $Pr\big(B(t) > b^{\varepsilon}_{\mathrm{net}}\big) \leq \varepsilon$, where

$$b^{\varepsilon}_{\mathrm{net}} = \inf_{s>0}\left\{\frac{1}{s}\big(\log \mathsf{M}_{\mathrm{net}}(s,t,t) - \log\varepsilon\big)\right\}$$

- DELAY: $Pr\big(W(t) > w^{\varepsilon}_{\mathrm{net}}\big) \leq \varepsilon$, where

$$\inf_{s>0}\left\{\mathsf{M}_{\mathrm{net}}(s,t+{w^{\varepsilon}}_{\mathrm{net}},t)\right\} \leq \varepsilon$$

Service Characterization: Information–Theoretic Model

- Channel capacity is characterized by [Tse and Hanly 1998] as

$$C(\mathbf{h},\mathbf{p})=\left\{\mathbf{r}:\sum_{i\in Q} r_i \leq W\log\left(1+\frac{\sum_{i\in Q}|h_i|^2 p_i}{N_0 W}\right), \forall Q\subseteq\{1,\ldots,L\}\right\}$$

  - $\mathbf{r},\mathbf{p}\in\mathbb{R}^L$ are the rate and power allocation vectors
  - $|h_i|^2$ is the channel gain for user i
  - Capacity limit is achieved by coding, e.g., successive decoding

- Assume m_n active users at a node n with equally allocated power among them
- $\mathcal{S}^{IT}_{n,j}$ is an SNR server for the through flow j at node n, where

$$\mathcal{S}^{IT}_{n,j}(\tau,t)=\prod_{u=\tau}^{t-1} g_j(\gamma_u)=\prod_{u=\tau}^{t-1}\left(1+\frac{1}{m_n(u)}\sum_{i=1}^{m_n(u)}\gamma_{i,u}\right)$$

  - This power scheme does not utilize user diversity

Service Characterization: Opportunistic Scheduler

- Maximizes throughput by scheduling user j at node n, s.t.

$$\gamma_{j,u} = \gamma_u^{\max} \triangleq \max\{\gamma_{i,u} : i = 1, \ldots, m_n\} \quad \forall u \geq 0$$

- Users with i.i.d. channel gains can access the channel $1/m_n$ of the time, then $\mathcal{S}_{n,j}^{OS}$ is a dynamic SNR server for user j, where

$$\mathcal{S}_{n,j}^{OS}(\tau, t) = \prod_{u=\tau}^{t-1} [g(\gamma_u^{\max})]^{1/m_n} \triangleq \prod_{u=\tau}^{t-1} g_j(\gamma_u^{\max})$$

- The distribution of $g_j(\gamma_u^{\max})$ is given by

$$F_{g_i(\gamma_u^{\max})}(x) = \left[F_{|h_i|^2}\left(\frac{x^{m_n} - 1}{\bar{\gamma}}\right)\right]^{m_n}, \quad x \geq 1$$

  - Where we used results from order statistics

Service Characterization: Random Access

- For the through flow j at node n, let $V_n(u)$ be the conditional virtual interference process during time slot u, then

$$V_n(u) = 1 - X_j(u) \cdot \prod_{i=1, i \neq j}^{m_n - 1} (1 - X_i(u))$$

  - $X_i(u), i \neq j$ are i.i.d. Bernoulli(p)
  - $X_j(u)$ is independent Bernoulli(p^*)
  - Then $V_n(u)$ is also Bernoulli($1 - q$)
  - $q = p^*(1-p)^{m_n - 1}$ $\equiv$ probability of successful transmission
  - We assume that all $X_i(u)$, and hence, $V_n(u)$, are stationary
- User j can transmit successfully at the channel capacity rate when $V_n(u) = 0$, hence, an SNR server for user j is given by

$$\mathcal{S}_{n,j}^{RA}(\tau, t) = \frac{\prod_{u=\tau}^{t-1} g(\gamma_u)}{\prod_{u=\tau}^{t-1} [g(\gamma_u)]^{V_n(u)}} = \prod_{u=\tau}^{t-1} [g(\gamma_u)]^{1 - V_n(u)}$$

  - the capacity offered to user j degenerates when $V_n(u) = 1$

Mellin Transform for Service Processes

- Bounds are in terms of Mellin transform of service processes
- For i.i.d. fading channels: $\mathcal{M}_{\mathcal{S}_{n,j}^{\text{any}}}(s,\tau,t) = \left[\mathcal{M}_{g_j(\gamma)}(s)\right]^{t-\tau}$
- Assuming Rayleigh fading and average channel gain $\bar{\gamma}$
- For the information–theoretic model, the Mellin transform is

$$\mathcal{M}_{g_j(\gamma)}(s) = \frac{1}{\Gamma(m_n)} \int_0^{\infty} x^{s-1} \left(\frac{m_n(x-1)}{\bar{\gamma}}\right)^{m_n-1} e^{-y} dx$$

- For the scheduling model, the Mellin transform is

$$\mathcal{M}_{g_i(\gamma_u^{\max})}(s) = \sum_{k=0}^{m_n-1} (-1)^k \binom{m_n-1}{k} \frac{m_n \bar{\gamma}^{\frac{s-1}{m_n}} e^{\frac{k+1}{\bar{\gamma}}}}{(k+1)^{\frac{s+m-1}{m_n}}} \Gamma\left(\frac{s+m_n-1}{m_n}, \frac{k+1}{\bar{\gamma}}\right)$$

- For the random access model, the Mellin transform is

$$\mathcal{M}_{\mathcal{S}_{n,i}^{RA}}(s,\tau,t) = \left[p(1-p)^{m_n-1} e^{\frac{1}{\bar{\gamma}}} \bar{\gamma}^{s-1} \Gamma(s, \bar{\gamma}^{-1})\right]^{t-\tau}$$

  - for $s > 1$ where $\Gamma(a,b)$ is the incomplete Gamma function

Performance Bounds of N Multiaccess Channels

- **Arrivals**: $(\sigma(s), \rho(s))$ bounded arrivals [Chang 2000]

$$\mathcal{M}_{\mathcal{A}}(s,\tau,t) \le e^{(s-1)\cdot(\rho(s-1)\cdot(t-\tau)+\sigma(s-1))}, \quad s > 1$$

- Define for the opportunistic scheduler model :

$$U(s,m) \triangleq \sum_{k=0}^{m-1}(-1)^k \binom{m-1}{k} \frac{m\bar{\gamma}^{\frac{-s}{m}} e^{s\rho(s)+\frac{k+1}{\bar{\gamma}}}}{(k+1)^{1-\frac{s}{m}}} \Gamma\left(\frac{m-s}{m}, \frac{k+1}{\bar{\gamma}}\right)$$

- BACKLOG: $Pr\left(B(t) > b_{\text{net}}^{\varepsilon}\right) \le \varepsilon$, where

$$b_{\text{net}}^{\varepsilon} = \inf_{s>0}\left\{\sigma(s) - \frac{1}{s}\left(N\log(1-U(s,m)) + \log\varepsilon\right)\right\}$$

- DELAY: $Pr\left(W(t) > w^{\varepsilon}\right) \le \varepsilon$, where

$$\inf_{s>0}\left\{\frac{e^{s(-\rho(s)w^{\varepsilon}+\sigma(s))}}{(1-U(s,m))^N} \cdot \min\left\{1, (U(s,m))^{w^{\varepsilon}}(w^{\varepsilon})^{N-1}\right\}\right\} \le \varepsilon$$

Performance Bounds of N Multiaccess Channels

- Define for the information–theoretic model and $m = 2$:

$$U^{IT}(s,2) = e^{s\rho(s)} \frac{1}{\Gamma(2)} e^{2/\bar{\gamma}} (\frac{\bar{\gamma}}{2})^{1-s} \cdot \Big(\Gamma(2-s, \frac{2}{\bar{\gamma}}) - \frac{2}{\bar{\gamma}}\Gamma(1-s, \frac{2}{\bar{\gamma}})\Big)$$

- Define for the random access model and $m = 2$:

$$U^{RA}(s,m) \triangleq e^{s\rho(s)} p^* (1-p)^{m-1} e^{\frac{1}{\bar{\gamma}}} \bar{\gamma}^{-s} \Gamma(1-s, \bar{\gamma}^{-1})$$

- Inserting U^{IT} and U^{RA} instead of U above gives bounds for the other two models
- for random number of active users $M_n(u)$, for any bound X

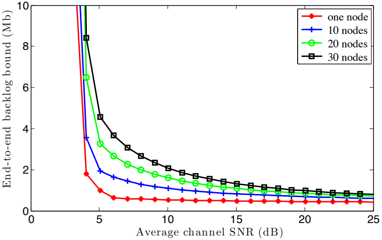
$$Pr(X(t) > x) = \sum_{m_n(0)=1}^{L} \cdots \sum_{m_n(t-1)=1}^{L} Pr(X(t) > x | \mathbf{M}_n = \mathbf{m}_n) Pr(\mathbf{M}_n = \mathbf{m}_n)$$

Numerical Results for N Rayleigh Channels

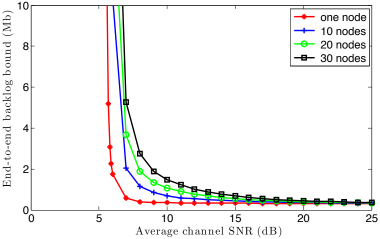
Model parameters

- $\Delta t = 1$ ms
- $W = 20$ kHz
- (σ, ρ) bounded traffic
- $\sigma = 50$ kb
- $\rho = 30$ kbps
- $\bar{\gamma} = 0$ to 40 dB
- $N = 1$ to 30
- $m = 100$
- $\varepsilon = 10^{-4}$

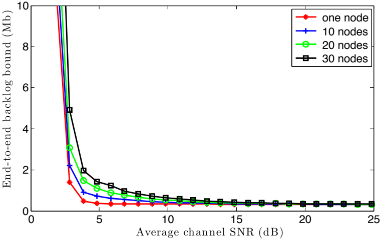
Backlog Bounds for N multiaccess Rayleigh Channels



Information–theoretic



Random access



Opportunistic scheduler

Conclusions

- A service characterization for three different multiaccess approaches
- Analysis in SNR domain using $(\min, \times)$ dioid algebra
  - Obtained end-to-end bounds of the three approaches
- Application to cascade of i.i.d. multiaccess channels with Rayleigh fading
  - Explicit bounds in terms of the physical channel parameters
  - Bounds scale linearly in N
- The analysis enables qualitative comparison of the end–to–end performance bounds under the three approaches

Thank you

Q & A