Some Min-Plus System Theory (1)

- Theorem (Concatenation of Service Elements – MGF Bound)

  Let there be two service elements in series. Assume the first element to be a dynamic S-server and the second element to be a dynamic T-server. Then holds:

  - The whole system is a dynamic $S \otimes T$-server.
  - If the dynamic servers are stochastically independent and bounded by $(\sigma_S(\theta), \rho_S(\theta))$ and $(\sigma_T(\theta), \rho_T(\theta))$, respectively, with $\rho_T(\theta), \rho_S(\theta) < 0$ and $\rho_S(\theta) \neq \rho_T(\theta)$ for some $\theta > 0$, the whole system is bounded by

$$(\sigma_S(\theta) + \sigma_T(\theta) + B, \max\{\rho_T(\theta), \rho_S(\theta)\})$$

$$\text{with: } B := -\frac{1}{\theta}\log(1 - e^{-\theta|\rho_S(\theta) - \rho_T(\theta)|})$$

- Dependent case: use Hölder‘s Iequality ...

Some Min-Plus System Theory (2)

- Theorem (Concatenation of Service Elements –Tail Bound)

  Let there be two service elements in series. Assume the first element to be a dynamic S-server and the second element to be a dynamic T-server. Then holds:

  - The whole system is a dynamic $S \otimes T$-server.
  - If the dynamic servers are tail-bounded by envelopes α_1 and α_2 with errors η_1 and η_2, respectively. Further assume $\alpha_1(n, \varepsilon)$ and $\alpha_2(n, \varepsilon)$ are monotonically increasing in n. Then the whole system is tail-bounded with envelope $\alpha_1 + \alpha_2$ and error $\eta_1 + \eta_2$.

- Remark: no independence assumption necessary!

Some Remarks on Service Models

- Defining tail-bounds directly on service process is less general
  - → See Michael‘s script

- Defining MGF bounds on the min-plus convolution of arrivals and service process is less general
  - → See Michael‘s script

- MGF bounded service is less general than tail-bounded service
  - Existence of MGF for S required
  - Independence assumption ... (jury is still out ...)