Distributed Computer Systems Lab
http://disco.informatik.uni-kl.de

Performance Modelling of Distributed Systems

5. Performance Bounds

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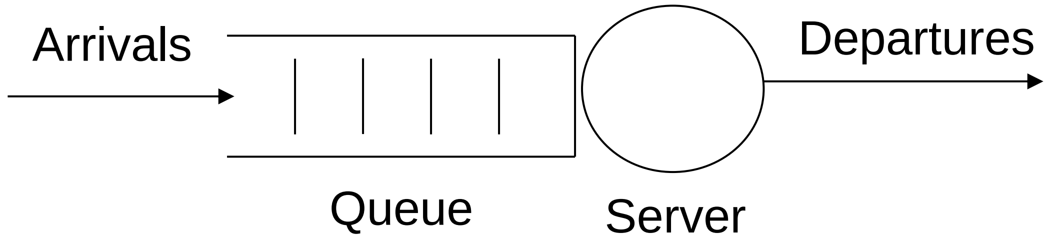
Performance Bounds



- Backlog
  - How much work is left in the system?
  - Buffer requirements
  - Vertical distance between arrival and departure flow(→ lossless system)
- Delay
  - How long does it take from entering queue until departure from server?
  - Horizontal distance between arrival and departure flow (→ lossless system and FIFO per flow assumption)I
- Output
  - Output description for further analysis

Backlog Bound (1)

- $\text{Backlog at time } n : q(n) := A(n) - D(n)$
- Some preliminaries
  - Bivariate (standard) convolution

$$f * g(m,n) = \sum_{k=m}^{n} f(m,k)g(k,n)$$

  - Bivariate (standard) deconvolution

$$f \circ g(m,n) = \sum_{k=0}^{m} f(k,n)g(k,m)$$

  - Bivariate (min,plus) deconvolution

$$A \oslash B(m,n) := \max_{0 \leq k \leq m} \{A(k,n) - B(k,m)\}$$

Backlog Bound (2)

- Theorem: (MGF Relation for (De-)Convolution)

  Let $X, Y \in \mathcal{S}$ be two stochastically independent random processes. Then it holds that:

  $$\phi_{X \otimes Y(m,n)}(-\theta) \leq \left(\phi_X(-\theta) * \phi_Y(-\theta)\right)(m,n)$$

  for all $\theta > 0$ and $(m,n) \in \Lambda(\mathbb{N}_0)$ such that the above MGFs exist.

  Further it holds that:

  $$\phi_{X \oslash Y(m,n)}(\theta) \leq \left(\phi_X(\theta) \circ \phi_Y(-\theta)\right)(m,n)$$

  for all $\theta > 0$ and $(m,n) \in \Lambda(\mathbb{N}_0)$ such that the above MGFs exist.

Backlog Bound (3)

- Theorem: (Backlog Bound under MGF Bounds)

  For the backlog $q(n)$ and all $x \in \mathbb{R}_0^+$ it holds that:

$$\mathbb{P}(q(n) > x) \leq e^{-\theta x + \theta(\sigma_A(\theta) + \sigma_S(\theta))} \sum_{k=0}^{n} e^{\theta k(\rho_A(\theta) + \rho_S(\theta))}.$$

- (Master) Strategy:
  1. Express $q(n)$ in terms of A and S
  2. Translate the resulting inequality in inequality of probabilities
  3. Use Chernoff bound to estimate tail probability
  4. Use MGF bounds of A and S

- Optimize bound after θ

Backlog Bound (4)

- Theorem: (Backlog Bound under Tail Bounds)

  For all $n \in \mathbb{N}_0$ and $\varepsilon > 0$ it holds that:

$$\mathbb{P}(q(n) > (\alpha_A(\varepsilon) + \alpha_S(\varepsilon)) \oslash S(n,n))$$
$$\leq \eta_S(n,\varepsilon) + \sum_{m=0}^{n} \eta_A(m,\varepsilon)$$

- Remarks
  - Finding the right ε for a given backlog bound may be non-trivial
  - We may use different ε for arrivals and service
  - Stochastic independence is nowhere assumed