Delay Bound (1)

- Definition: (Virtual Delay)

  The virtual delay d at time n is defined as:

$$d(n) := \min\{m : A(n) < D(n+m)\}.$$

- Theorem (Delay Bound under MGF Bounds)

  For all $N \in \mathbb{N}_0$ it holds that:

$$\mathbb{P}(d(n) > N) \leq e^{\theta\rho_S(\theta)N + \theta(\sigma_A(\theta)+\sigma_S(\theta))} \sum_{k=0}^{n+N} e^{\theta(n-k)(\rho_A(\theta)+\rho_S(\theta))}$$

- Can and should be optimized for θ
- Corollary: (Simplified Form)

  Assume $S(k, n+N) \geq 0 \quad \forall k \geq n+1,$ then:

$$\mathbb{P}(d(n) > N) \leq \inf_{\theta \in [0,b]} e^{\theta\rho_S(\theta)N + \theta(\sigma_A(\theta)+\sigma_S(\theta))} \sum_{k=0}^{n} e^{\theta k(\rho_A(\theta)+\rho_S(\theta))}$$

Delay Bound (2)

- Definition: (Delay-Stability)

We say S is delay-stable (for ε), iff

$$S(m,n) - \alpha_S(n-m,\varepsilon) \geq 0 \quad \forall\, m,n \geq 0$$
$$S(m,n) - \alpha_S(n-m,\varepsilon) \xrightarrow{n\to\infty} \infty \qquad \forall\, m \geq 0$$

- Theorem: (Delay Bound under Tail Bounds)

Assume S is delay stable for some ε. Then for all $n \in \mathbb{N}_0$:

$$\mathbb{P}(d(n) > \min\{N \geq 0 : (\alpha_S(\varepsilon) - S) \oslash \alpha_A(\varepsilon)(n, n+N') < 0\, \forall\, N' \geq N)$$
$$\leq \sum_{k=0}^{n} \eta_A(k,\varepsilon) + \sum_{k=n+1}^{\infty} \eta_S(k,\varepsilon)$$