Excercise Sheet 3

Performance Modeling of Distributed Systems

Winter Term 2013/2014

Assignment 1 (A Token Bucket)
Consider a flow with the following properties: The first increment $a(1)$ is stochastically independent of all other increments and further exponentially distributed with parameter λ. For all other increments holds $a(i) \leq r$ for some rate r. Compute a $\sigma(\theta), \rho(\theta)$-bound and derive the maximal value θ can achieve for that bound. Further derive a tail-bound. (Hint for the tailbound: Consider $a(i) = r$ first.)

Assignment 2 (Multiplexing Tailbounds)
Assume two flows A, B, which are both tailbounded, i.e.:

$$\mathbb{P}(A(n) - A(m) > \alpha(n - m, \varepsilon)) \leq \eta(n - m, \varepsilon)$$

and

$$\mathbb{P}(B(n) - B(m) > \beta(n - m, \varepsilon)) \leq \zeta(n - m, \varepsilon)$$

Derive a tailbound for the multiplexed flow, defined by: $A \oplus B(n) = A(n) + B(n)$.

Assignment 3 (Quality of Markov- and Chernoff-bounds)
Assume a random variable X, which is exponentially distributed with parameter λ (or binomially distributed with parameters p, n). Bound the probability $\mathbb{P}(X > x)$ using Chernoff's and Markov's bound (optimize over the parameter θ in Chernoff's bound!). Plot the results for varying λ (or varying p and fixed n) and compare them, to the actual probability.

Assignment 4 (Acuity)
We now investigate the parameter θ in the $(\sigma(\theta), \rho(\theta))$-bounds. Generate some (in the order of thousands) exponentially distributed random variables with parameter $\lambda = 10$ (you might reuse your code from previous exercises). Sort your realisations by magnitude and plot them. Now apply the function $f(x) = e^{\theta x}$ to your realisations with varying parameter $\theta \in [0, 10)$. How does the plot change for different values of θ?

We ask now how many of the realisations are larger than the expected value of one of the realisations $\mathbb{E}(e^{\theta X})$. For this we already know $\mathbb{E}(e^{\theta X}) = \frac{\lambda}{\lambda - \theta}$. Write a procedure, which counts for a fixed θ the number of realisations being larger $\frac{\lambda}{\lambda - \theta}$. What percentage of your realisations is larger for a small θ? What is it for a large θ? Explain the difference with the help of your previously produced plots!