Excercise Sheet 4

Performance Modeling of Distributed Systems

Winter Term 2013/2014

Assignment 1 (Concatenation breaks Strictness)
Assume $S_i(m,n) = S_i(n) - S_i(m)$ for $i \in \{1,2\}$. Give an example for S_i and $m \leq n$, such that

$$S(m,n) := S_1 \otimes S_2(m,n) \neq S(0,n) - S(0,m)$$

Assignment 2 (A Semi-Stochastic Backlog Bound)
Assume a flow A to be $\sigma_A(\theta), \rho_A(\theta)$-bounded. Further assume that the corresponding departures at some server fulfill: $D(n) \geq A \otimes \beta(n)$. Show that the following backlog bound holds:

$$\mathbb{P}(q(n) > x) \leq e^{-\theta x} \sum_{k=0}^{n} e^{\theta \rho_A(\theta)(n-k) + \theta \sigma_A(\theta) - \theta \beta(n-k)}$$

Assignment 3 (Cross-Flows break Strictness)
In this exercise we investigate how *strict* a server can be, after subtracting a cross-flow from it. Throughout this we assume a node serving two arrivals, of which A_1 is prioritized over A_2.

- Assume first the service element is a dynamic S-Server with respect to $A := A_1 + A_2$. Show that it is also a dynamic S_l-Server with respect to A_2 with $S_l(m,n) = S(m,n) - A_1(m,n)$.
- Assume now the service element fulfills Lindley's equation with respect to A. Construct input flows A_1, A_2 and a service S such that for some interval $[m,n]$ with $D_2(m-1) = A_2(m-1)$ holds $S_l(m,n) > D_2(m,n)$, i.e. S_l is not a strict service curve. [1] (Hint: You can choose a very simple S)

[1] In fact any backlogged interval would work here. But in taking the additional assumption $D_2(m-1) = A_2(m-1)$ into account, we can also exclude that the leftover service could be *weak strict*. A dynamic S-Server is defined to be *weak strict* if for any backlogged period $[m,n]$ with $D(m-1) = A(m-1)$ holds: $D(m,n) \geq S(m,n)$.