Network Calculus Tests – Single Server Network Configurations

Version 1.1 (2014-Dec-30)

General Information

- The network calculus analyses presented in this document were created for the purpose of testing the Disco Deterministic Network Calculator (DiscoDNC)[1] – an open-source deterministic network calculus tool developed by the *Distributed Computer Systems (DISCO) Lab* at the University of Kaiserslautern.
- Naming of the individual network configurations depicts the name of the according functional test for the DiscoDNC.
- The naming scheme used in this document is detailed in NetworkCalculus_NamingScheme.pdf.
- Arrival bound computations are equivalent to the `PbooArrivalBound_Output_PerHop.java` class of the DiscoDNC.
- The end-to-end left-over service curve for PBOO arrival bounds can be computed by simply convolving the server-local ones.
- Arrival bounds for `PmooArrivalBound.java` and analyses using them are listed only if results are different to PBOO.

Changelog:

Version 1.1 (2014-Dec-30):

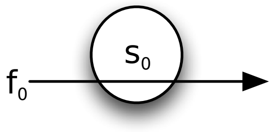
- Adapted to naming scheme version 1.1.

Acknowledgements:

Version 1.1: Thanks to Yokanand Thirupathi and Paresh Chotala for pointing out some errors.

[1] http://disco.cs.uni-kl.de/index.php/projects/disco-dnc

Single_1Flow



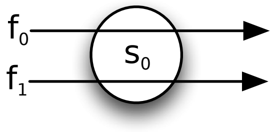
- $\beta_{s_0} = \beta_{R_{s_0},T_{s_0}} = \beta_{10,10}$
- $\mathcal{F} = \{f_0\}$
- $\alpha^{f_0} = \gamma_{r^{f_0},b^{f_0}} = \gamma_{5,25}$

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha^{f_0}$	$= \gamma_{5,25}$	
	D^{f_0}	$\begin{aligned} \beta_{s_0} &= b_{s_o} \\ 10 \cdot [t-10]^+ &= 25 \\ t &= 12\frac{1}{2} \end{aligned}$	FIFO per micro flow $\begin{aligned} \beta_{s_0} &= b_{s_o} \\ 10 \cdot [t-10]^+ &= 25 \\ t &= 12\frac{1}{2} \end{aligned}$
	B^{f_0}	$\begin{aligned} \alpha_{s_0}(T_{s_0}) &= 5 \cdot 10 + 25 \\ &= 75 \end{aligned}$	

SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} = [\beta_{s_0} - \alpha_{s_0}^{x(f_0)}]^+ = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0},T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}} = \beta_{s_0}$	$= \beta_{10,10}$	
	D^{f_0}	$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} &= b^{f_0} \\ 10 \cdot [t-10]^+ &= 25 \\ t &= 12\frac{1}{2} \end{aligned}$	
	B^{f_0}	$\begin{aligned} \alpha^{f_0}(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}) &= 5 \cdot 10 + 25 \\ &= 75 \end{aligned}$	

<table>
<tr><th colspan="2">PMOO</th><th>ARB_MUX</th></tr>
<tr><td rowspan="2">s_0</td><td>$\alpha_{s_0}^{\bar{x}(f_0)}$</td><td>$= \gamma_{0,0}$</td></tr>
<tr><td>$\alpha_{s_0}^{x(f_0)}$</td><td>$= \gamma_{0,0}$</td></tr>
<tr><td rowspan="3">$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}$</td><td>$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} = R_{s_0} - r_{s_0}^{x(f_0)}$</td><td>$\begin{aligned} &= 10 - 0 \\ &= 10 \end{aligned}$</td></tr>
<tr><td>$T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} = T_{s_0} + \frac{b_{s_0}^{\bar{x}(f_0)} + r_{s_0}^{x(f_0)} \cdot T_{s_0}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}$</td><td>$\begin{aligned} &= 10 + \frac{0 + 0 \cdot 10}{10} \\ &= 10 \end{aligned}$</td></tr>
<tr><td>$=$</td><td>$= \beta_{10,10}$</td></tr>
<tr><td colspan="2">D^{f_0}</td><td>$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} &= b^{f_0} \\ 10 \cdot [t - 10]^+ &= 25 \\ t &= 12\frac{1}{2} \end{aligned}$</td></tr>
<tr><td colspan="2">B^{f_0}</td><td>$\begin{aligned} \alpha^{f_0}(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}) &= 5 \cdot 10 + 25 \\ &= 75 \end{aligned}$</td></tr>
</table>

Single_2Flows_1AC



- $\beta_{s_0} = \beta_{R_{s_0}, T_{s_0}} = \beta_{10,10}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}$, $n \in \{0, 1\}$

Flows f_n, $n \in \{0, 1\}$

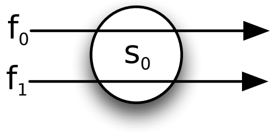
TFA results will be equal for all flows as they share the same path of servers.

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha^{f_0} + \alpha^{f_1}$	$= \gamma_{10,50}$	
	D^{f_n}	$\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 50$ $t = 15$	$\beta_{s_0} = \alpha_{s_0}$ $10 \cdot [t - 10]^+ = 10 \cdot t + 50$ $0 \cdot t = 150$ $\Rightarrow D^{f_n} = \infty$
	B^{f_n}	$\alpha_{s_0}(T_{s_0}) = 10 \cdot 10 + 50$ $= 150$	

	SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_n)} = \alpha^{f_n}$		$= \gamma_{5,25}$	
	$\beta_{s_0}^{\mathrm{l.o.}f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_n)} = \beta_{R_{s_0}^{\mathrm{l.o.}f_n}, T_{s_0}^{\mathrm{l.o.}f_n}}$	$R_{s_0}^{\mathrm{l.o.}f_n}$	$[R_{s_0} - r_{s_0}^{x(f_n)}]^+ = 5$	
		$T_{s_0}^{\mathrm{l.o.}f_n}$	$\begin{aligned} \beta_{s_0} &= b_{s_0}^{x(f_n)} \\ 10 \cdot [t-10]^+ &= 25 \\ t &= 12\frac{1}{2} \end{aligned}$	$\begin{aligned} \beta_{s_0} &= \alpha_{s_0}^{x(f_n)} \\ 10 \cdot [t-10]^+ &= 5 \cdot t + 25 \\ t &= 25 \end{aligned}$
		$=$	$=\beta_{5,12\frac{1}{2}}$	$=\beta_{5,25}$
	$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_n} = \beta_{s_0}^{\mathrm{l.o.}f_n}$		$= \beta_{5,12\frac{1}{2}}$	$= \beta_{5,25}$
	D^{f_n}		$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_n} &= b^{f_n} \\ 5 \cdot [t - 12\frac{1}{2}]^+ &= 25 \\ t &= 17\frac{1}{2} \end{aligned}$	$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_n} &= b^{f_n} \\ 5 \cdot [t-25]^+ &= 25 \\ t &= 30 \end{aligned}$
	B^{f_n}		$\begin{aligned} \alpha^{f_n}(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_n}) &= 5 \cdot 12\frac{1}{2} + 25 \\ &= 87\frac{1}{2} \end{aligned}$	$\begin{aligned} \alpha^{f_n}(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_n}) &= 5 \cdot 25 + 25 \\ &= 150 \end{aligned}$

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}(f_n)} = \alpha^{f_n}$	$= \gamma_{5,25}$
	$\alpha_{s_0}^{x(f_n)} = \alpha^{f_n}$	$= \gamma_{5,25}$
$\beta_{\text{e2e}}^{\text{l.o.}f_n} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_n}, T_{\text{e2e}}^{\text{l.o.}f_n}}$	$R_{\text{e2e}}^{\text{l.o.}f_n} = R_{s_0} - r_{s_0}^{x(f_n)}$	$\begin{aligned} &= 10 - 5 \\ &= 5 \end{aligned}$
	$T_{\text{e2e}}^{\text{l.o.}f_n} = T_{s_0} + \frac{b_{s_0}^{\bar{x}(f_n)} + r_{s_0}^{x(f_n)} \cdot T_{s_0}}{R_{\text{e2e}}^{\text{l.o.}f_0}}$	$\begin{aligned} &= 10 + \frac{25 + 5 \cdot 10}{5} \\ &= 25 \end{aligned}$
	$=$	$= \beta_{5,25}$
D^{f_n}		$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_n} &= b^{f_n} \\ 5 \cdot [t - 25]^{+} &= 25 \\ t &= 30 \end{aligned}$
B^{f_n}		$\begin{aligned} \alpha^{f_n}(T_{\text{e2e}}^{\text{l.o.}f_n}) &= 5 \cdot 25 + 25 \\ &= 150 \end{aligned}$

Single_2Flow_2ACs



- $\beta_{s_0} = \beta_{R_{s_0},T_{s_0}} = \beta_{10,10}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \gamma_{r^{f_0},b^{f_0}} = \gamma_{4,10}$
- $\alpha^{f_1} = \gamma_{r^{f_1},b^{f_1}} = \gamma_{5,25}$

Flows f_n, $n \in \{0,1\}$

TFA results will be equal for all flows as they share the same path of servers.

	TFA	FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha^{f_0} + \alpha^{f_1}$	$= \gamma_{9,35}$	
	D^{f_n}	$\begin{aligned} \beta_{s_0} &= b_{s_0} \\ 10 \cdot [t-10]^+ &= 35 \\ t &= 13\frac{1}{2} \end{aligned}$	$\begin{aligned} \beta_{s_0} &= \alpha_{s_0} \\ 10 \cdot [t-10]^+ &= 9 \cdot t + 35 \\ t &= 135 \end{aligned}$
	B^{f_n}	$\begin{aligned} \alpha_{s_0}(T_{s_0}) &= 9 \cdot 10 + 35 \\ &= 125 \end{aligned}$	

Flow f_0

SFA				FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)} = \alpha^{f_1}$			$= \gamma_{5,25}$	
	$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$	$R_{s_0}^{\text{l.o.}f_0}$		$[R_{s_0} - r_{s_0}^{x(f_0)}]^+ = 5$	
		$T_{s_0}^{\text{l.o.}f_0}$		$\begin{aligned} \beta_{s_0} &= b_{s_0}^{x(f_0)} \\ 10 \cdot [t-10]^+ &= 25 \\ t &= 12\frac{1}{2} \end{aligned}$	$\begin{aligned} \beta_{s_0} &= \alpha_{s_0}^{x(f_0)} \\ 10 \cdot [t-10]^+ &= 5 \cdot t + 25 \\ t &= 25 \end{aligned}$
		$=$		$=\beta_{5,12\frac{1}{2}}$	$=\beta_{5,25}$
	$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}} = \beta_{s_0}^{\text{l.o.}f_0}$			$=\beta_{5,12\frac{1}{2}}$	$= \beta_{5,25}$
	D^{f_0}			$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_0} &= b^{f_0} \\ 5 \cdot [t - 12\frac{1}{2}]^+ &= 10 \\ t &= 14\frac{1}{2} \end{aligned}$	$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_0} &= b^{f_0} \\ 5 \cdot [t-25]^+ &= 10 \\ t &= 27 \end{aligned}$
	B^{f_0}			$\begin{aligned} \alpha^{f_0}(T_{\text{e2e}}^{\text{l.o.}f_0}) &= 4 \cdot 12\frac{1}{2} + 10 \\ &= 60 \end{aligned}$	$\begin{aligned} \alpha^{f_0}(T_{\text{e2e}}^{\text{l.o.}f_0}) &= 4 \cdot 25 + 10 \\ &= 110 \end{aligned}$

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}(f_0)} = \alpha^{f_1}$	$= \gamma_{5,25}$
	$\alpha_{s_0}^{x(f_0)} = \alpha^{f_1}$	$= \gamma_{5,25}$
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$	$R_{\text{e2e}}^{\text{l.o.}f_0} = R_{s_0} - r_{s_0}^{x(f_0)}$	$\begin{aligned} &= 10 - 5 \\ &= 5 \end{aligned}$
	$T_{\text{e2e}}^{\text{l.o.}f_0} = T_{s_0} + \frac{b_{s_0}^{\bar{x}(f_0)} + r_{s_0}^{x(f_0)} \cdot T_{s_0}}{R_{\text{e2e}}^{\text{l.o.}f_0}}$	$\begin{aligned} &= 10 + \frac{25 + 5 \cdot 10}{5} \\ &= 25 \end{aligned}$
	$=$	$= \beta_{5,25}$
D^{f_0}		$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_0} &= b^{f_0} \\ 5 \cdot [t - 25]^+ &= 10 \\ t &= 27 \end{aligned}$
B^{f_0}		$\begin{aligned} \alpha^{f_0}(T_{\text{e2e}}^{\text{l.o.}f_0}) &= 4 \cdot 25 + 10 \\ &= 110 \end{aligned}$

Flow f_1

	SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_1)} = \alpha^{f_0}$		$= \gamma_{4,10}$	
	$\beta_{s_0}^{\text{l.o.}f_1} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_1)} = \beta_{R_{s_0}^{\text{l.o.}f_1}, T_{s_0}^{\text{l.o.}f_1}}$	$R_{s_0}^{\text{l.o.}f_1}$	$[R_{s_0} - r_{s_0}^{x(f_1)}]^+ = \quad 6$	
		$T_{s_0}^{\text{l.o.}f_1}$	$\beta_{s_0} = b_{s_0}^{x(f_1)}$ $10 \cdot [t-10]^+ = 10$ $t = 11$	$\beta_{s_0} = \alpha_{s_0}^{x(f_1)}$ $10 \cdot [t-10]^+ = 4 \cdot t + 10$ $t = 18\frac{1}{3}$
		$=$	$=\beta_{6,11}$	$=\beta_{6,18\frac{1}{3}}$
	$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f_1}} = \beta_{s_0}^{\text{l.o.}f_1}$		$=\beta_{6,11}$	$= \beta_{6,18\frac{1}{3}}$
	D^{f_1}		$\beta_{\text{e2e}}^{\text{l.o.}f_1} = b^{f_1}$ $6 \cdot [t-11]^+ = 25$ $t = 15\frac{1}{6}$	$\beta_{\text{e2e}}^{\text{l.o.}f_1} = b^{f_1}$ $6 \cdot [t - 18\frac{1}{3}]^+ = 25$ $t = 22\frac{1}{2}$
	B^{f_1}		$\alpha^{f_1}(T_{\text{e2e}}^{\text{l.o.}f_1}) = 5 \cdot 11 + 25$ $= 80$	$\alpha^{f_1}(T_{\text{e2e}}^{\text{l.o.}f_1}) = 5 \cdot 18\frac{1}{3} + 25$ $= 116\frac{2}{3}$

<table>
<tr><td colspan="2">PMOO</td><td>ARB_MUX</td></tr>
<tr><td rowspan="2">s_0</td><td>$\alpha_{s_0}^{\bar{x}(f_1)} = \alpha^{f_0}$</td><td>$= \gamma_{4,10}$</td></tr>
<tr><td>$\alpha_{s_0}^{x(f_1)} = \alpha^{f_0}$</td><td>$= \gamma_{4,10}$</td></tr>
<tr><td rowspan="3">$\beta_{s_0}^{\text{l.o.}f_1} = \beta_{R_{s_0}^{\text{l.o.}f_1}, T_{s_0}^{\text{l.o.}f_1}}$</td><td>$R_{\text{e2e}}^{\text{l.o.}f_1} = R_{s_0} - r_{s_0}^{x(f_1)}$</td><td>$\begin{aligned} &= 10-4 \\ &= 6 \end{aligned}$</td></tr>
<tr><td>$T_{\text{e2e}}^{\text{l.o.}f_1} = T_{s_0} + \frac{b_{s_0}^{\bar{x}(f_1)} + r_{s_0}^{x(f_1)} \cdot T_{s_0}}{R_{\text{e2e}}^{\text{l.o.}f_0}}$</td><td>$\begin{aligned} &= 10 + \frac{10 + 4 \cdot 10}{6} \\ &= 18\frac{1}{3} \end{aligned}$</td></tr>
<tr><td>$=$</td><td>$= \beta_{6,18\frac{1}{3}}$</td></tr>
<tr><td colspan="2">D^{f_1}</td><td>$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_1} &= b^{f_1} \\ 6 \cdot [t - 18\frac{1}{3}]^+ &= 25 \\ t &= 22\frac{1}{2} \end{aligned}$</td></tr>
<tr><td colspan="2">B^{f_1}</td><td>$\begin{aligned} \alpha^{f_1}(T_{\text{e2e}}^{\text{l.o.}f_1}) &= 5 \cdot 18\frac{1}{3} + 25 \\ &= 116\frac{2}{3} \end{aligned}$</td></tr>
</table>

Single_10Flow_10ACs

- $\beta_{s_0} = \beta_{R_{s_0},T_{s_0}} = \beta_{10,10}$
- $\mathcal{F} = \{f_0, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9\}$
- for $n = 0$ to 9: $\alpha^{f_n} = \gamma_{r^{f_n},b^{f_n}} = \gamma_{\frac{1}{10}\cdot(i+1),1\cdot(i+1)}$

We restrict the presentation of the SFA and the PMOO analysis to flows f_0 and f_6. The omitted computations follow the same scheme.

Flows f_n, $n \in \{0, \ldots, 9\}$

TFA results will be equal for all flows as they share the same path of servers.

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \sum_{n=0}^{9} \alpha_i$	$= \gamma_{5\frac{1}{2},55}$	
	D^{f_n}	$\beta_{s_0} = b_{s_0}$ $10 \cdot [t-10]^+ = 55$ $t = 15\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}$ $10 \cdot [t-10]^+ = 5\frac{1}{2} \cdot t + 55$ $t = 34\frac{4}{9}$
	B^{f_n}	$\alpha_{s_0}(T_{s_0}) = 5\frac{1}{2} \cdot 10 + 55$ $= 110$	

Flow f_0

	SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)} = \sum_{n=1}^{9} \alpha^{f_n} = \gamma_{r_{s_0}^{x(f_0)}, b_{s_0}^{x(f_0)}}$	$r_{s_0}^{x(f_0)}$	$\sum_{n=1}^{9} r^{f_n} = 5\frac{2}{5}$	
		$b_{s_0}^{x(f_0)}$	$\sum_{n=1}^{9} b^{f_n} = 54$	
		$=$	$=\gamma_{5\frac{2}{5},54}$	
	$\beta_{s_0}^{\text{l.o.}f_0} = R_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$	$R_{s_0}^{\text{l.o.}f_0}$	$[R_{s_0} - r_{s_0}^{x(f_0)}]^+ = 4\frac{3}{5}$	
		$T_{s_0}^{\text{l.o.}f_0}$	$\begin{aligned} \beta_{s_0} &= b_{s_0}^{x(f_0)} \\ 10 \cdot [t-10]^+ &= 54 \\ t &= 15\frac{2}{5} \end{aligned}$	$\begin{aligned} \beta_{s_0} &= \alpha_{s_0}^{x(f_0)} \\ 10 \cdot [t-10]^+ &= 5\frac{2}{5} \cdot t + 54 \\ t &= 33\frac{11}{23} \end{aligned}$
		$=$	$=\beta_{4\frac{3}{5},15\frac{2}{5}}$	$=\beta_{4\frac{3}{5},33\frac{11}{23}}$
	$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}} = \beta_{s_0}^{\text{l.o.}f_0}$		$=\beta_{4\frac{3}{5},15\frac{2}{5}}$	$= \beta_{4\frac{3}{5},33\frac{11}{23}}$
	D^{f_0}		$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_0} &= b^{f_0} \\ 4\frac{3}{5} \cdot [t - 15\frac{2}{5}]^+ &= 1 \\ t &= 15\frac{71}{115} \end{aligned}$	$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_0} &= b^{f_0} \\ 4\frac{3}{5} \cdot [t - 33\frac{11}{23}]^+ &= 1 \\ t &= 33\frac{16}{23} \end{aligned}$
	B^{f_0}		$\begin{aligned} \alpha^{f_0}(T_{\text{e2e}}^{\text{l.o.}f_0}) &= \frac{1}{10} \cdot 15\frac{2}{5} + 1 \\ &= 2\frac{27}{50} \end{aligned}$	$\begin{aligned} \alpha^{f_0}(T_{\text{e2e}}^{\text{l.o.}f_0}) &= \frac{1}{10} \cdot 33\frac{11}{23} + 1 \\ &= 4\frac{8}{23} \end{aligned}$

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}(f_0)} = \sum_{n=1}^{9} \alpha^{f_n} = \gamma_{r_{s_0}^{x(f_0)}, b_{s_0}^{x(f_0)}}$	$=\gamma_{5\frac{2}{5},54}$
	$\alpha_{s_0}^{x(f_0)} = \sum_{n=1}^{9} \alpha^{f_n} = \gamma_{r_{s_0}^{x(f_0)}, b_{s_0}^{x(f_0)}}$	$=\gamma_{5\frac{2}{5},54}$
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$	$R_{\text{e2e}}^{\text{l.o.}f_0} = R_{s_0} - r_{s_0}^{x(f_0)}$	$= \quad 10 - 5\frac{2}{5}$ $= \quad 4\frac{3}{5}$
	$T_{\text{e2e}}^{\text{l.o.}f_0} = T_{s_0} + \frac{b_{s_0}^{\bar{x}(f_0)} + r_{s_0}^{x(f_0)} \cdot T_{s_0}}{R_{\text{e2e}}^{\text{l.o.}f_0}}$	$= \quad 10 + \frac{54 + 5\frac{2}{5} \cdot 10}{4\frac{3}{5}}$ $= \quad 10 + \frac{108}{4\frac{3}{5}}$ $= \quad 33\frac{11}{23}$
	$=$	$= \beta_{4\frac{3}{5}, 33\frac{11}{23}}$
D^{f_0}		$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \quad b^{f_0}$ $4\frac{3}{5} \cdot [t - 33\frac{11}{23}]^+ = \quad 1$ $t = \quad 33\frac{16}{23}$
B^{f_0}		$\alpha^{f_0}(T_{\text{e2e}}^{\text{l.o.}f_0}) = \quad \frac{1}{10} \cdot 33\frac{11}{23} + 1$ $= \quad 4\frac{8}{23}$

Flow f_6

	SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha^{x(f_6)} = \sum_{n=0}^{5} \alpha^{f_n} + \sum_{n=7}^{9} \alpha^{f_n} = \gamma_{r_{s_0}^{x(f_6)}, b_{s_0}^{x(f_6)}}$	$r_{s_0}^{x(f_6)}$	$(\sum_{n=0}^{9} r^{f_n}) - r^{f_6} = 4\frac{4}{5}$	
		$b_{s_0}^{x(f_6)}$	$(\sum_{n=0}^{9} b^{f_n}) - b^{f_6} = 48$	
		$=$	$=\gamma_{4\frac{4}{5},48}$	
	$\beta_{s_0}^{\mathrm{l.o.}f_6} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_6)} = \beta_{R_{s_0}^{\mathrm{l.o.}f_6}, T_{s_0}^{\mathrm{l.o.}f_6}}$	$R_{s_0}^{\mathrm{l.o.}f_6}$	$[R_{s_0} - r_{s_0}^{x(f_6)}]^{+} = 5\frac{1}{5}$	
		$T_{s_0}^{\mathrm{l.o.}f_6}$	$\begin{aligned} \beta_{s_0} &= b_{s_0}^{x(f_6)} \\ 10 \cdot [t-10]^{+} &= 48 \\ t &= 14\frac{4}{5} \end{aligned}$	$\begin{aligned} \beta_{s_0} &= \alpha_{s_0}^{x(f_6)} \\ 10 \cdot [t-10]^{+} &= 4\frac{4}{5} \cdot t + 48 \\ t &= 28\frac{6}{13} \end{aligned}$
		$=$	$=\beta_{5\frac{1}{5},14\frac{4}{5}}$	$=\beta_{5\frac{1}{5},28\frac{6}{13}}$
	$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_6} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_6}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_6}}$		$=\beta_{5\frac{1}{5},14\frac{4}{5}}$	$= \beta_{5\frac{1}{5},28\frac{6}{13}} = \beta_{s_0}^{\mathrm{l.o.}f_6}$
	D^{f_6}		$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_6} &= b^{f_6} \\ 5\frac{1}{5} \cdot [t - 14\frac{4}{5}]^{+} &= 7 \\ t &= 16\frac{19}{130} \end{aligned}$	$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_6} &= b^{f_6} \\ 5\frac{1}{5} \cdot [t - 28\frac{6}{13}]^{+} &= 7 \\ t &= 29\frac{21}{26} \end{aligned}$
	B^{f_6}		$\begin{aligned} \alpha^{f_6}(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_6}) &= \frac{7}{10} \cdot 14\frac{4}{5} + 7 \\ &= 17\frac{9}{25} \end{aligned}$	$\begin{aligned} \alpha^{f_6}(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_6}) &= \frac{7}{10} \cdot 28\frac{6}{13} + 7 \\ &= 26\frac{12}{13} \end{aligned}$

PMOO		ARB_MUX
s_0	$\alpha^{\bar{x}(f_6)} = \sum_{n=0}^{5} \alpha^{f_n} + \sum_{n=7}^{9} \alpha^{f_n} = \gamma_{r_{s_0}^{x(f_6)}, b_{s_0}^{x(f_6)}}$	$=\gamma_{4\frac{4}{5},48}$
	$\alpha^{x(f_6)} = \sum_{n=0}^{5} \alpha^{f_n} + \sum_{n=7}^{9} \alpha^{f_n} = \gamma_{r_{s_0}^{x(f_6)}, b_{s_0}^{x(f_6)}}$	$=\gamma_{4\frac{4}{5},48}$
$\beta_{s_0}^{\text{l.o.}f_6} = \beta_{R_{s_0}^{\text{l.o.}f_6}, T_{s_0}^{\text{l.o.}f_6}}$	$R_{\text{e2e}}^{\text{l.o.}f_6} = R_{s_0} - r_{s_0}^{x(f_6)}$	$\begin{aligned} &= 10 - 4\frac{4}{5} \\ &= 5\frac{1}{5} \end{aligned}$
	$T_{\text{e2e}}^{\text{l.o.}f_6} = T_{s_0} + \frac{b_{s_0}^{\bar{x}(f_6)} + r_{s_0}^{x(f_6)} \cdot T_{s_0}}{R_{\text{e2e}}^{\text{l.o.}f_0}}$	$\begin{aligned} &= 10 + \frac{48 + 4\frac{4}{5} \cdot 10}{5\frac{1}{5}} \\ &= 10 + \frac{96}{5\frac{1}{5}} \\ &= 28\frac{6}{13} \end{aligned}$
	$=$	$=\beta_{5\frac{1}{5},28\frac{6}{13}}$
D^{f_6}		$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_6} &= b^{f_6} \\ 5\frac{1}{5} \cdot [t - 28\frac{6}{13}]^{+} &= 7 \\ t &= 29\frac{21}{26} \end{aligned}$
B^{f_6}		$\begin{aligned} \alpha^{f_6}(T_{\text{e2e}}^{\text{l.o.}f_6}) &= \frac{7}{10} \cdot 28\frac{6}{13} + 7 \\ &= 26\frac{12}{13} \end{aligned}$