Network Calculus Tests – Feed Forward Network Settings

Version 2.0 beta (2015-Jul-11)

General Information

- The network calculus analyses presented in this document were created for the purpose of testing the Disco Deterministic Network Calculator (DiscoDNC)[1] – an open-source deterministic network calculus tool developed by the *Distributed Computer Systems (DISCO) Lab* at the University of Kaiserslautern.
- Naming of the individual network settings depicts the name of the according functional test for the DiscoDNC.
- The naming scheme used in this document is detailed in NetworkCalculus_NamingScheme.pdf.
- Arrival bounds for `PmooArrivalBound.java` and analyses using them are listed only if results differ from `PbooArrivalBound_Concatenation.java`.

Changelog:

Version 1.1 (2014-Dec-30):

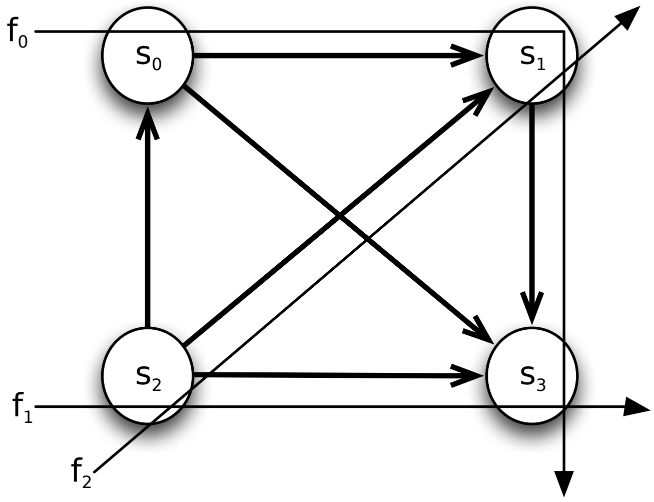
- Streamlined the PMOO left-over latency $T_{\mathrm{e2e}}^{\mathrm{l.o.}f}$ computation.
- Adaption to naming scheme version 1.1.

Version 2.0 beta (2015-Jul-11):

- Rework of Arrival Bounds documentation
  - Parameters: see DiscoDNC's computeArrivalBounds(Server server, Set<Flow> flows_to_bound, Flow flow_of_interest).
  - Bounding arrivals moved to the analysis requiring the specific bounds if they differ between flows of interest (may cause duplication).
  - The algebraic derivations is included within many tabular bounding procedures. They are adapted to `PbooArrivalBound_Concatenation.java`, yet, in contrast to the current DiscoDNC code, they may reuse known results.
- The naming scheme was slightly updated to include sets of servers $\mathbb{S}$ and sets of Flows $\mathbb{F}$.
- Minor consistency fixes for variable names.

[1] http://disco.cs.uni-kl.de/index.php/projects/disco-dnc

FeedForward_1SC_3Flows_1AC_3Paths



$\mathbb{S} = \{s_0, s_1, s_2, s_3\}$ with

$$\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{s_3} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20},\ i \in \{0, 1, 2\}$$

$\mathbb{F} = \{f_0, f_1, f_2\}$ with

$$\alpha^{f_0} = \alpha^{f_1} = \alpha^{f_2} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25},\ n \in \{0, 1, 2\}$$

Flow f_0

Total Flow Analysis

Arrival Bounds

<table>
<tr><td colspan="2">$(s_1, \{f_0\}, \emptyset) =: \alpha_{s_1}^{f_0}$</td><td>FIFO Multiplexing</td><td>Arbitrary Multiplexing</td></tr>
<tr><td colspan="4">$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0}$</td></tr>
<tr><td colspan="2">$\alpha_{s_0}^{x(f_0)}$</td><td colspan="2">$= \gamma_{0,0}$</td></tr>
<tr><td colspan="2">$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$
$= \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$</td><td colspan="2">$= \beta_{s_0} = \beta_{20,20}$</td></tr>
<tr><td rowspan="3">$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0}$
$= \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$</td><td>$r_{s_1}^{f_0}$</td><td colspan="2">$= 5$</td></tr>
<tr><td>$b_{s_1}^{f_0}$</td><td colspan="2">$= \alpha^{f_0}\left(T^{\text{l.o.}}\right)$
$= 5 \cdot 20 + 25$
$= 125$</td></tr>
<tr><td>$=$</td><td colspan="2">$= \gamma_{5,125}$</td></tr>
</table>

<table>
<tr><td colspan="2">$(s_3, \{f_1\}, \emptyset) =: \alpha_{s_3}^{f_1}$
$(s_1, \{f_2\}, \emptyset) =: \alpha_{s_1}^{f_2}$
$=: \alpha_{s_i}^{f_n}$ with $(n, i) \in \{(1,3), (2,1)\}$</td><td>FIFO Multiplexing</td><td>Arbitrary Multiplexing</td></tr>
<tr><td colspan="4">$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n}$</td></tr>
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<tr><td rowspan="3">$\beta_{s_2}^{\text{l.o.}f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_n)}$
$= \beta_{R_{s_2}^{\text{l.o.}f_n}, T_{s_2}^{\text{l.o.}f_n}}$</td><td>$R_{s_2}^{\text{l.o.}f_n}$</td><td colspan="2">$\left[R_{s_2} - r_{s_2}^{x(f_n)}\right]^+ = 20 - 5$
$= 15$</td></tr>
<tr><td>$T_{s_2}^{\text{l.o.}f_n}$</td><td>$\beta_{s_2} = b_{s_2}^{x(f_n)}$
$20 \cdot [t-20]^+ = 25$
$t = 21\frac{1}{4}$</td><td>$\beta_{s_2} = \alpha_{s_2}^{x(f_n)}$
$20 \cdot [t-20]^+ = 5 \cdot t + 25$
$t = 28\frac{1}{3}$</td></tr>
<tr><td>$=$</td><td>$= \beta_{15,21\frac{1}{4}}$</td><td>$= \beta_{15,28\frac{1}{3}}$</td></tr>
<tr><td rowspan="3">$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n}$
$= \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}}$</td><td>$r_{s_i}^{f_n}$</td><td colspan="2">$= 5$</td></tr>
<tr><td>$b_{s_i}^{f_n}$</td><td>$= \alpha^{f_n}(T_{s_2}^{\text{l.o.}f_n})$
$= 5 \cdot 21\frac{1}{4} + 25$
$= 131\frac{1}{4}$</td><td>$= \alpha^{f_n}(T_{s_2}^{\text{l.o.}f_n})$
$= 5 \cdot 28\frac{1}{3} + 25$
$= 166\frac{2}{3}$</td></tr>
<tr><td>$=$</td><td>$= \gamma_{5,131\frac{1}{4}}$</td><td>$= \gamma_{5,166\frac{2}{3}}$</td></tr>
</table>

$(s_3, \{f_0\}, \emptyset) =: \alpha_{s_3}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_3}^{f_0} = \alpha^{f_0} \oslash \left(\beta_{s_0}^{\mathrm{l.o.}f_0} \otimes \beta_{s_1}^{\mathrm{l.o.}f_0}\right)$ (reuse of previous result) $= \alpha^{f_0} \oslash \beta_{s_0}^{\mathrm{l.o.}f_0} \oslash \beta_{s_1}^{\mathrm{l.o.}f_0} = \alpha_{s_1}^{f_0} \oslash \beta_{s_1}^{\mathrm{l.o.}f_0}$			
$\alpha_{s_1}^{x(f_0)}$		$= \alpha_{s_1}^{f_2} = \gamma_{5,131\frac{1}{4}}$	$= \alpha_{s_1}^{f_2} = \gamma_{5,166\frac{2}{3}}$
$\beta_{s_1}^{\mathrm{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)} = \beta_{R_{s_1}^{\mathrm{l.o.}f_0}, T_{s_1}^{\mathrm{l.o.}f_0}}$	$R_{s_1}^{\mathrm{l.o.}f_0}$	$\left[R_{s_1} - r_{s_1}^{x(f_0)}\right]^+ = 20 - 5 = 15$	
	$T_{s_1}^{\mathrm{l.o.}f_0}$	$\beta_{s_1} = b_{s_1}^{x(f_0)}$ $20 \cdot [t-20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$ $20 \cdot [t-20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{8}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{8}}$
$\alpha_{s_1}^{f_0}$		$= \gamma_{5,125}$	
$\alpha_{s_3}^{f_0} = \alpha^{f_0} \oslash \beta_{s_1}^{\mathrm{l.o.}f_0} = \gamma_{r_{s_3}^{f_0}, b_{s_3}^{f_0}}$	$r_{s_3}^{f_0}$	$= 5$	
	$b_{s_3}^{f_0}$	$= \alpha_{s_1}^{f_0}\left(T_{s_1}^{\mathrm{l.o.}f_0}\right)$ $= 5 \cdot 26\frac{9}{16} + 125$ $= 257\frac{13}{16}$	$= \alpha_{s_1}^{f_0}\left(T_{s_1}^{\mathrm{l.o.}f_0}\right)$ $= 5 \cdot 37\frac{7}{8} + 125$ $= 313\frac{8}{9}$
	$=$	$= \gamma_{5,257\frac{13}{16}}$	$= \gamma_{5,313\frac{8}{9}}$

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing
s_0	α_{s_0}	$= \alpha^{f_0}_{s_0} = \gamma_{5,25}$	
	$D^{f_0}_{s_0}$	$\begin{aligned} \beta_{s_0} &= b_{s_0} \\ 20 \cdot [t-20]^+ &= 25 \\ t &= 21\frac{1}{4} \end{aligned}$	FIFO per micro flow $\begin{aligned} \beta_{s_0} &= b_{s_0} \\ 20 \cdot [t-20]^+ &= 25 \\ t &= 21\frac{1}{4} \end{aligned}$
	$B^{f_0}_{s_0}$	$\begin{aligned} \alpha_{s_0}(T_{s_0}) &= 20 \cdot 5 + 25 \\ &= 125 \end{aligned}$	
s_1	α_{s_1}	$\begin{aligned} &= \alpha^{f_0}_{s_1} + \alpha^{f_2}_{s_1} \\ &= \gamma_{5,125} + \gamma_{5,131\frac{1}{4}} \\ &= \gamma_{10,256\frac{1}{4}} \end{aligned}$	$\begin{aligned} &= \alpha^{f_0}_{s_1} + \alpha^{f_2}_{s_1} \\ &= \gamma_{5,125} + \gamma_{5,166\frac{2}{3}} \\ &= \gamma_{10,291\frac{2}{3}} \end{aligned}$
	$D^{f_0}_{s_1}$	$\begin{aligned} \beta_{s_1} &= b_{s_1} \\ 20 \cdot [t-20]^+ &= 256\frac{1}{4} \\ t &= 32\frac{13}{16} \end{aligned}$	$\begin{aligned} \beta_{s_1} &= \alpha_{s_1} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 291\frac{2}{3} \\ t &= 69\frac{1}{6} \end{aligned}$
	$B^{f_0}_{s_1}$	$\begin{aligned} \alpha_{s_1}(T_{s_1}) &= 10 \cdot 20 + 256\frac{1}{4} \\ &= 456\frac{1}{4} \end{aligned}$	$\begin{aligned} \alpha_{s_1}(T_{s_1}) &= 10 \cdot 20 + 291\frac{2}{3} \\ &= 491\frac{2}{3} \end{aligned}$
s_3	α_{s_3}	$\begin{aligned} &= \alpha^{f_0}_{s_3} + \alpha^{f_1}_{s_3} \\ &= \gamma_{5,257\frac{13}{16}} + \gamma_{5,131\frac{1}{4}} \\ &= \gamma_{10,389\frac{1}{16}} \end{aligned}$	$\begin{aligned} &= \alpha^{f_0}_{s_3} + \alpha^{f_1}_{s_3} \\ &= \gamma_{5,313\frac{8}{9}} + \gamma_{5,166\frac{2}{3}} \\ &= \gamma_{10,480\frac{5}{9}} \end{aligned}$
	$D^{f_0}_{s_3}$	$\begin{aligned} \beta_{s_3} &= b_{s_3} \\ 20 \cdot [t-20]^+ &= 389\frac{1}{16} \\ t &= 39\frac{29}{64} \end{aligned}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 480\frac{5}{9} \\ t &= 88\frac{5}{90} \end{aligned}$
	$B^{f_0}_{s_3}$	$\begin{aligned} \alpha_{s_3}(T_{s_3}) &= 10 \cdot 20 + 389\frac{1}{16} \\ &= 589\frac{1}{16} \end{aligned}$	$\begin{aligned} \alpha_{s_3}(T_{s_3}) &= 10 \cdot 20 + 480\frac{5}{9} \\ &= 680\frac{5}{9} \end{aligned}$
D^{f_0}		$\begin{aligned} &= \sum_{i=\{0,1,3\}} D^{f_0}_{s_i} \\ &= 93\frac{33}{64} \end{aligned}$	$\begin{aligned} &= \sum_{i=\{0,1,3\}} D^{f_0}_{s_i} \\ &= 178\frac{17}{36} \end{aligned}$
B^{f_0}		$\begin{aligned} &= \max_{i=\{0,1,3\}} B^{f_0}_{s_i} \\ &= 589\frac{1}{16} \end{aligned}$	$\begin{aligned} &= \max_{i=\{0,1,3\}} B^{f_0}_{s_i} \\ &= 680\frac{5}{9} \end{aligned}$

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$(s_3, \{f_1\}, f_0) =: \alpha^{f_1}_{s_3}$ $(s_1, \{f_2\}, f_0) =: \alpha^{f_2}_{s_1}$ $=: \alpha^{f_n}_{s_i}$ with $(n,i) \in \{(1,3),(2,1)\}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_n}_{s_i} = \alpha^{f_n} \oslash \beta^{\text{l.o.}f_n}_{s_2}$			
$\alpha^{x(f_n)}_{s_2}$		$= \gamma_{5,25}$	
$\beta^{\text{l.o.}f_n}_{s_2} = \beta_{s_2} \ominus \alpha^{x(f_n)}_{s_2}$ $= \beta_{R^{\text{l.o.}f_n}_{s_2}, T^{\text{l.o.}f_n}_{s_2}}$	$R^{\text{l.o.}f_n}_{s_2}$	$\left[R_{s_2} - r^{x(f_n)}_{s_2}\right]^+ = 20 - 5$ $= 15$	
	$T^{\text{l.o.}f_n}_{s_2}$	$\beta_{s_2} = b^{x(f_n)}_{s_2}$ $20 \cdot [t-20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_2} = \alpha^{x(f_n)}_{s_2}$ $20 \cdot [t-20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
$\alpha^{f_n}_{s_i} = \alpha^{f_n} \oslash \beta^{\text{l.o.}f_n}_{s_2}$ $= \gamma_{r^{f_n}_{s_i}, b^{f_n}_{s_i}}$	$r^{f_n}_{s_i}$	$= 5$	
	$b^{f_n}_{s_i}$	$= \alpha^{f_n}\left(T^{\text{l.o.}f_n}_{s_2}\right)$ $= 5 \cdot 21\frac{1}{4} + 25$ $= 131\frac{1}{4}$	$= \alpha^{f_n}\left(T^{\text{l.o.}f_n}_{s_2}\right)$ $= 5 \cdot 28\frac{1}{3} + 25$ $= 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$

Remark:
In this network setting, we have $(s_3, \{f_1\}, f_0) = (s_3, \{f_1\}, \emptyset)$ and $(s_1, \{f_2\}, f_0) = (s_1, \{f_2\}, \emptyset)$ because neither (cross-)flow f_1 nor f_2 interferes with the flow of interest f_0 on multiple consecutive hops.

Analyses

	SFA		FIFO Multiplexing	Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_0)}$		$= \gamma_{0,0}$	
	$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$		$= \beta_{20,20}$	
s_1	$\alpha_{s_1}^{x(f_0)}$		$= \alpha_{s_1}^{f_2} = \gamma_{5,131\frac{1}{4}}$	$= \alpha_{s_1}^{f_2} = \gamma_{5,166\frac{2}{3}}$
	$\begin{aligned} \beta_{s_1}^{\text{l.o.}f_0} &= \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)} \\ &= \beta_{R_{s_1}^{\text{l.o.}f_0}, T_{s_1}^{\text{l.o.}f_0}} \end{aligned}$	$R_{s_1}^{\text{l.o.}f_0}$	$\begin{aligned} \left[R_{s_1} - r_{s_1}^{x(f_0)}\right]^+ &= 20 - 5 \\ &= 15 \end{aligned}$	
		$T_{s_1}^{\text{l.o.}f_0}$	$\begin{aligned} \beta_{s_1} &= b_{s_1}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 131\frac{1}{4} \\ t &= 26\frac{9}{16} \end{aligned}$	$\begin{aligned} \beta_{s_1} &= \alpha_{s_1}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 166\frac{2}{3} \\ t &= 37\frac{7}{9} \end{aligned}$
		$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{9}}$
s_3	$\alpha_{s_3}^{x(f_0)}$		$= \alpha_{s_3}^{f_1} = \gamma_{5,131\frac{1}{4}}$	$= \alpha_{s_3}^{f_1} = \gamma_{5,166\frac{2}{3}}$
	$\begin{aligned} \beta_{s_3}^{\text{l.o.}f_0} &= \beta_{s_3} \ominus \alpha_{s_3}^{x(f_0)} \\ &= \beta_{R_{s_3}^{\text{l.o.}f_0}, T_{s_3}^{\text{l.o.}f_0}} \end{aligned}$	$R_{s_3}^{\text{l.o.}f_0}$	$\begin{aligned} \left[R_{s_3} - r_{s_3}^{x(f_0)}\right]^+ &= 20 - 5 \\ &= 15 \end{aligned}$	
		$T_{s_3}^{\text{l.o.}f_0}$	$\begin{aligned} \beta_{s_3} &= b_{s_3}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 131\frac{1}{4} \\ t &= 26\frac{9}{16} \end{aligned}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 166\frac{2}{3} \\ t &= 37\frac{7}{9} \end{aligned}$
		$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{9}}$
$\beta_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0} = \beta_{R_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0}, T_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0}}$			$\begin{aligned} &= \bigotimes_{i=\{0,1,3\}} \beta_{s_i}^{\text{l.o.}f_0} \\ &= \beta_{15,73\frac{1}{8}} \end{aligned}$	$\begin{aligned} &= \bigotimes_{i=\{0,1,3\}} \beta_{s_i}^{\text{l.o.}f_0} \\ &= \beta_{15,95\frac{5}{9}} \end{aligned}$
D^{f_0}			$\begin{aligned} \beta_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0} &= b^{f_0} \\ 15 \cdot \left[t - 73\frac{1}{8}\right]^+ &= 25 \\ t &= 74\frac{19}{24} \end{aligned}$	$\begin{aligned} \beta_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0} &= b^{f_0} \\ 15 \cdot \left[t - 95\frac{5}{9}\right]^+ &= 25 \\ t &= 97\frac{2}{9} \end{aligned}$
B^{f_0}			$\begin{aligned} \alpha^{f_0}\left(T_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0}\right) &= 5 \cdot 73\frac{1}{8} + 25 \\ &= 390\frac{5}{8} \end{aligned}$	$\begin{aligned} \alpha^{f_0}\left(T_{\langle s_0,s_1,s_3\rangle}^{\text{l.o.}f_0}\right) &= 5 \cdot 95\frac{5}{9} + 25 \\ &= 502\frac{7}{9} \end{aligned}$

PMOO		Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_0}^{\bar{x}(f_0)}$	
s_1	$\alpha_{s_1}^{x(f_0)}$	$= \alpha_{s_1}^{f_2} = \gamma_{5,166\frac{2}{3}}$
	$\alpha_{s_1}^{\bar{x}(f_0)}$	
s_3	$\alpha_{s_3}^{x(f_0)}$	$= \alpha_{s_3}^{f_1} = \gamma_{5,166\frac{2}{3}}$
	$\alpha_{s_3}^{\bar{x}(f_0)}$	
$\beta_{\langle s_0,s_1,s_3\rangle}^{\mathrm{l.o.}f_0} = \beta_{R_{\langle s_0,s_1,s_3\rangle}^{\mathrm{l.o.}f_0},T_{\langle s_0,s_1,s_3\rangle}^{\mathrm{l.o.}f_0}}$	$R_{\langle s_0,s_1,s_3\rangle}^{\mathrm{l.o.}f_0}$	$= \bigwedge_{i\in\{0,1,3\}} \left(R_{s_i} - r_{s_i}^{x(f_0)}\right)$ $= (20-5) \wedge (20-5) \wedge (20-5)$ $= 15$
	$T_{\langle s_0,s_1,s_3\rangle}^{\mathrm{l.o.}f_0}$	$= \sum_{i\in\{0,1,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}\right)$ $= 20 + \frac{0+0\cdot 20}{15} + 20 + \frac{166\frac{2}{3}+5\cdot 20}{15} + 20 + \frac{166\frac{2}{3}+5\cdot 20}{15}$ $= 60 + \frac{533\frac{1}{3}}{15}$ $= 95\frac{5}{9}$
	$=$	$= \beta_{15,95\frac{5}{9}}$
D^{f_0}		$\beta_{\langle s_0,s_1,s_3\rangle}^{\mathrm{l.o.}f_0} = b^{f_0}$ $15 \cdot \left[t - 95\frac{5}{9}\right]^+ = 25$ $t = 97\frac{2}{9}$
B^{f_0}		$\alpha^{f_0}\left(T_{\langle s_0,s_1,s_3\rangle}^{\mathrm{l.o.}f_0}\right) = 5 \cdot 95\frac{5}{9} + 25$ $= 502\frac{7}{9}$

Flow f_1

Total Flow Analysis

Arrival Bounds

$\begin{aligned}(s_3,\{f_1\},\emptyset) &=: \alpha^{f_1}_{s_3}\\ (s_1,\{f_2\},\emptyset) &=: \alpha^{f_2}_{s_1}\\ =: \alpha^{f_n}_{s_i} \text{ with } (n,i) &\in \{(1,3),(2,1)\}\end{aligned}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_n}_{s_i} = \quad \alpha^{f_n} \oslash \beta^{\mathrm{l.o.}f_n}_{s_2}$			
$\alpha^{x(f_n)}_{s_2}$		$= \gamma_{5,25}$	
$\begin{aligned}\beta^{\mathrm{l.o.}f_n}_{s_2} &= \beta_{s_2} \ominus \alpha^{x(f_n)}_{s_2}\\ &= \beta_{R^{\mathrm{l.o.}f_n}_{s_2},T^{\mathrm{l.o.}f_n}_{s_2}}\end{aligned}$	$R^{\mathrm{l.o.}f_n}_{s_2}$	$\begin{aligned}\left[R_{s_2} - r^{x(f_n)}_{s_2}\right]^+ &= 20-5\\ &= 15\end{aligned}$	
	$T^{\mathrm{l.o.}f_n}_{s_2}$	$\begin{aligned}\beta_{s_2} &= b^{x(f_n)}_{s_2}\\ 20\cdot[t-20]^+ &= 25\\ t &= 21\frac{1}{4}\end{aligned}$	$\begin{aligned}\beta_{s_2} &= \alpha^{x(f_n)}_{s_2}\\ 20\cdot[t-20]^+ &= 5\cdot t+25\\ t &= 28\frac{1}{3}\end{aligned}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
$\begin{aligned}\alpha^{f_n}_{s_i} &= \alpha^{f_n} \oslash \beta^{\mathrm{l.o.}f_n}_{s_2}\\ &= \gamma_{r^{f_n}_{s_i},b^{f_n}_{s_i}}\end{aligned}$	$r^{f_n}_{s_i}$	$= 5$	
	$b^{f_n}_{s_i}$	$\begin{aligned}&= \alpha^{f_n}\left(T^{\mathrm{l.o.}f_n}_{s_2}\right)\\ &= 5\cdot 21\frac{1}{4}+25\\ &= 131\frac{1}{4}\end{aligned}$	$\begin{aligned}&= \alpha^{f_n}\left(T^{\mathrm{l.o.}f_n}_{s_2}\right)\\ &= 5\cdot 28\frac{1}{3}+25\\ &= 166\frac{2}{3}\end{aligned}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$

$(s_3,\{f_0\},\emptyset) =: \alpha^{f_0}_{s_3}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_0}_{s_3} = \quad \alpha^{f_0} \oslash \left(\beta^{\mathrm{l.o.}f_0}_{s_0} \otimes \beta^{\mathrm{l.o.}f_0}_{s_1}\right)$			
$\alpha^{x(f_0)}_{s_0}$		$= \gamma_{0,0}$	
$\begin{aligned}\beta^{\mathrm{l.o.}f_0}_{s_0} &= \beta_{s_0} \ominus \alpha^{x(f_0)}_{s_0}\\ &= \beta_{R^{\mathrm{l.o.}f_0}_{s_0},T^{\mathrm{l.o.}f_0}_{s_0}}\end{aligned}$		$= \beta_{s_0} = \beta_{20,20}$	
$\alpha^{x(f_0)}_{s_1}$		$= \alpha^{f_2}_{s_1} = \gamma_{5,131\frac{1}{4}}$	$= \alpha^{f_2}_{s_1} = \gamma_{5,166\frac{2}{3}}$
$\begin{aligned}\beta^{\mathrm{l.o.}f_0}_{s_1} &= \beta_{s_1} \ominus \alpha^{x(f_0)}_{s_1}\\ &= \beta_{R^{\mathrm{l.o.}f_0}_{s_1},T^{\mathrm{l.o.}f_0}_{s_1}}\end{aligned}$	$R^{\mathrm{l.o.}f_0}_{s_1}$	$\begin{aligned}\left[R_{s_1} - r^{x(f_0)}_{s_1}\right]^+ &= 20-5\\ &= 15\end{aligned}$	
	$T^{\mathrm{l.o.}f_0}_{s_1}$	$\begin{aligned}\beta_{s_1} &= b^{x(f_0)}_{s_1}\\ 20\cdot[t-20]^+ &= 131\frac{1}{4}\\ t &= 26\frac{9}{16}\end{aligned}$	$\begin{aligned}\beta_{s_1} &= \alpha^{x(f_0)}_{s_1}\\ 20\cdot[t-20]^+ &= 5\cdot t+166\frac{2}{3}\\ t &= 37\frac{7}{8}\end{aligned}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{8}}$
$\beta^{\mathrm{l.o.}f_0}_{s_0} \otimes \beta^{\mathrm{l.o.}f_0}_{s_1} = \beta^{\mathrm{l.o.}f_0}_{\langle s_0,s_1\rangle}$		$\begin{aligned}&= \beta_{20,20} \otimes \beta_{15,26\frac{9}{16}}\\ &= \beta_{15,46\frac{9}{16}}\end{aligned}$	$\begin{aligned}&= \beta_{20,20} \otimes \beta_{15,37\frac{7}{8}}\\ &= \beta_{15,57\frac{7}{8}}\end{aligned}$
$\alpha^{f_0}_{s_3} = \alpha^{f_0} \oslash \left(\beta^{\mathrm{l.o.}f_0}_{s_0} \otimes \beta^{\mathrm{l.o.}f_0}_{s_1}\right)$	$r^{f_0}_{s_3}$	$= 5$	
	$b^{f_0}_{s_3}$	$\begin{aligned}&= \alpha^{f_0}_{s_1}\left(T^{\mathrm{l.o.}f_0}_{\langle s_0,s_1\rangle}\right)\\ &= 5\cdot 46\frac{9}{16}+25\\ &= 257\frac{13}{16}\end{aligned}$	$\begin{aligned}&= \alpha^{f_0}_{s_1}\left(T^{\mathrm{l.o.}f_0}_{\langle s_0,s_1\rangle}\right)\\ &= 5\cdot 57\frac{7}{8}+25\\ &= 313\frac{8}{9}\end{aligned}$
	$=$	$= \gamma_{5,257\frac{13}{16}}$	$= \gamma_{5,313\frac{8}{9}}$

Remark:
`PmooArrivalBound.java` will have the same result as `PbooArrivalBound_Concatenation.java` because f_0 does not have cross-traffic interfering on multiple consecutive hops.

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing
s_2	α_{s_2}	$= \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2}$ $= \gamma_{5,25} + \gamma_{5,25}$ $= \gamma_{10,50}$	
	$D_{s_2}^{f_1}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t-20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t-20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{s_2}^{f_1}$	$\alpha_{s_2}(T_{s_2}) = 20 \cdot 10 + 50$ $= 250$	
s_3	α_{s_3}	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$ $= \gamma_{5,257\frac{13}{16}} + \gamma_{5,131\frac{1}{4}}$ $= \gamma_{10,389\frac{1}{16}}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1}$ $= \gamma_{5,313\frac{8}{9}} + \gamma_{5,166\frac{2}{3}}$ $= \gamma_{10,480\frac{5}{9}}$
	$D_{s_3}^{f_1}$	$\beta_{s_3} = b_{s_3}$ $20 \cdot [t-20]^+ = 389\frac{1}{16}$ $t = 39\frac{29}{64}$	$\beta_{s_3} = \alpha_{s_3}$ $20 \cdot [t-20]^+ = 10 \cdot t + 480\frac{5}{9}$ $t = 88\frac{5}{90}$
	$B_{s_3}^{f_1}$	$\alpha_{s_3}(T_{s_3}) = 10 \cdot 20 + 389\frac{1}{16}$ $= 589\frac{1}{16}$	$\alpha_{s_3}(T_{s_3}) = 10 \cdot 20 + 480\frac{5}{9}$ $= 680\frac{5}{9}$
D^{f_1}		$= \sum_{i=2}^{3} D_{s_i}^{f_1}$ $= 61\frac{61}{64}$	$= \sum_{i=2}^{3} D_{s_i}^{f_1}$ $= 185\frac{5}{9}$
B^{f_1}		$= \max_{i=\{2,3\}} B_{s_i}^{f_1}$ $= 589\frac{1}{16}$	$= \max_{i=\{2,3\}} B_{s_i}^{f_1}$ $= 680\frac{5}{9}$

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$(s_1, \{f_2\}, \emptyset) =: \alpha^{f_2}_{s_1}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_2}_{s_1} = \alpha^{f_2} \oslash \beta^{\text{l.o.}f_2}_{s_2}$			
$\alpha^{x(f_2)}_{s_2}$		$= \gamma_{5,25}$	
$\beta^{\text{l.o.}f_2}_{s_2} = \beta_{s_2} \ominus \alpha^{x(f_2)}_{s_2} = \beta_{R^{\text{l.o.}f_2}_{s_2}, T^{\text{l.o.}f_2}_{s_2}}$	$R^{\text{l.o.}f_n}_{s_2}$	$\left[R_{s_2} - r^{x(f_2)}_{s_2}\right]^+ = 20 - 5 = 15$	
	$T^{\text{l.o.}f_n}_{s_2}$	$\beta_{s_2} = b^{x(f_2)}_{s_2}$ $20 \cdot [t-20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_2} = \alpha^{x(f_2)}_{s_2}$ $20 \cdot [t-20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
$\alpha^{f_2}_{s_1} = \alpha^{f_2} \oslash \beta^{\text{l.o.}f_2}_{s_2} = \gamma_{r^{f_2}_{s_1}, b^{f_2}_{s_1}}$	$r^{f_2}_{s_1}$	$= 5$	
	$b^{f_2}_{s_1}$	$= \alpha^{f_2}\left(T^{\text{l.o.}f_2}_{s_2}\right)$ $= 5 \cdot 21\frac{1}{4} + 25$ $= 131\frac{1}{4}$	$= \alpha^{f_2}\left(T^{\text{l.o.}f_2}_{s_2}\right)$ $= 5 \cdot 28\frac{1}{3} + 25$ $= 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$

$(s_3, \{f_0\}, \emptyset) =: \alpha^{f_0}_{s_3}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_0}_{s_3} = \alpha^{f_0} \oslash \left(\beta^{\text{l.o.}f_0}_{s_0} \otimes \beta^{\text{l.o.}f_0}_{s_1}\right)$			
$\alpha^{x(f_0)}_{s_0}$		$= \gamma_{0,0}$	
$\beta^{\text{l.o.}f_0}_{s_0} = \beta_{s_0} \ominus \alpha^{x(f_0)}_{s_0} = \beta_{R^{\text{l.o.}f_0}_{s_0}, T^{\text{l.o.}f_0}_{s_0}}$		$= \beta_{s_0} = \beta_{20,20}$	
$\alpha^{x(f_0)}_{s_1}$		$= \alpha^{f_2}_{s_1} = \gamma_{5,131\frac{1}{4}}$	$= \alpha^{f_2}_{s_1} = \gamma_{5,166\frac{2}{3}}$
$\beta^{\text{l.o.}f_0}_{s_1} = \beta_{s_1} \ominus \alpha^{x(f_0)}_{s_1} = \beta_{R^{\text{l.o.}f_0}_{s_1}, T^{\text{l.o.}f_0}_{s_1}}$	$R^{\text{l.o.}f_0}_{s_1}$	$\left[R_{s_1} - r^{x(f_0)}_{s_1}\right]^+ = 20 - 5 = 15$	
	$T^{\text{l.o.}f_0}_{s_1}$	$\beta_{s_1} = b^{x(f_0)}_{s_1}$ $20 \cdot [t-20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{s_1} = \alpha^{x(f_0)}_{s_1}$ $20 \cdot [t-20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{8}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{8}}$
$\beta^{\text{l.o.}f_0}_{s_0} \otimes \beta^{\text{l.o.}f_0}_{s_1} = \beta^{\text{l.o.}f_0}_{\langle s_0, s_1\rangle}$		$= \beta_{20,20} \otimes \beta_{15,26\frac{9}{16}}$ $= \beta_{15,46\frac{9}{16}}$	$= \beta_{20,20} \otimes \beta_{15,37\frac{7}{8}}$ $= \beta_{15,57\frac{7}{8}}$
$\alpha^{f_0}_{s_3} = \alpha^{f_0} \oslash \left(\beta^{\text{l.o.}f_0}_{s_0} \otimes \beta^{\text{l.o.}f_0}_{s_1}\right)$	$r^{f_0}_{s_3}$	$= 5$	
	$b^{f_0}_{s_3}$	$= \alpha^{f_0}_{s_1}\left(T^{\text{l.o.}f_0}_{\langle s_0, s_1\rangle}\right)$ $= 5 \cdot 46\frac{9}{16} + 25$ $= 257\frac{13}{16}$	$= \alpha^{f_0}_{s_1}\left(T^{\text{l.o.}f_0}_{\langle s_0, s_1\rangle}\right)$ $= 5 \cdot 57\frac{7}{8} + 25$ $= 313\frac{8}{9}$
	$=$	$= \gamma_{5,257\frac{13}{16}}$	$= \gamma_{5,313\frac{8}{9}}$

Remark:
`PmooArrivalBound.java` will have the same result as `PbooArrivalBound_Concatenation.java` because f_0 does not have cross-traffic interfering on multiple consecutive hops.

Analyses

SFA			FIFO Multiplexing	Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_1)}$		$= \alpha^{f_2} = \gamma_{5,25}$	
	$\beta_{s_2}^{\text{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)}$ $= \beta_{R_{s_2}^{\text{l.o.}f_1}, T_{s_2}^{\text{l.o.}f_1}}$		$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s_3	$\alpha_{s_3}^{x(f_1)} = \alpha_{s_3}^{f_0}$		$= \gamma_{5,257\frac{13}{16}}$	$= \gamma_{5,313\frac{8}{9}}$
	$\beta_{s_3}^{\text{l.o.}f_1} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_1)}$ $= \beta_{R_{s_3}^{\text{l.o.}f_1}, T_{s_3}^{\text{l.o.}f_1}}$	$R_{s_3}^{\text{l.o.}f_1}$	$\left[R_{s_3} - r_{s_3}^{x(f_1)}\right]^+ = 20 - 5$ $= 15$	
		$T_{s_3}^{\text{l.o.}f_1}$	$\beta_{s_3} = b_{s_3}^{x(f_1)}$ $20 \cdot [t-20]^+ = 257\frac{13}{16}$ $t = 32\frac{57}{64}$	$\beta_{s_3} = \alpha_{s_3}^{x(f_1)}$ $20 \cdot [t-20]^+ = 5 \cdot t + 313\frac{8}{9}$ $t = 47\frac{16}{27}$
		$=$	$= \beta_{15,32\frac{57}{64}}$	$= \beta_{15,47\frac{16}{27}}$
$\beta_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1} = \beta_{R_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}, T_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}}$			$= \bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.}f_1}$ $= \beta_{15,54\frac{9}{64}}$	$= \bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.}f_1}$ $= \beta_{15,75\frac{25}{27}}$
D^{f_1}			$\beta_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1} = b^{f_1}$ $15 \cdot \left[t - 54\frac{9}{64}\right]^+ = 25$ $t = 55\frac{155}{192}$	$\beta_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1} = b^{f_1}$ $15 \cdot \left[t - 75\frac{25}{27}\right]^+ = 25$ $t = 77\frac{16}{27}$
B^{f_1}			$\alpha^{f_1}\left(T_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}\right) = 5 \cdot 54\frac{9}{64} + 25$ $= 295\frac{45}{64}$	$\alpha^{f_1}\left(T_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}\right) = 5 \cdot 75\frac{25}{27} + 25$ $= 404\frac{17}{27}$

PMOO		Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_1)}$ $\alpha_{s_2}^{\bar{x}(f_1)}$	$= \alpha_{s_2}^{f_2} = \gamma_{5,25}$
s_3	$\alpha_{s_3}^{x(f_1)}$ $\alpha_{s_3}^{\bar{x}(f_1)}$	$= \alpha_{s_3}^{f_0} = \gamma_{5,313\frac{8}{9}}$
$\beta_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1} = \beta_{R_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}, T_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}}$	$R_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}$	$= \bigwedge_{i\in\{2,3\}} \left(R_{s_i} - r_{s_i}^{x(f_1)}\right)$ $= (20-5) \wedge (20-5)$ $= 15$
	$T_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}$	$= \sum_{i\in\{2,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{\text{e2e}}^{\text{l.o.}f_1}}\right)$ $= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{313\frac{8}{9} + 5 \cdot 20}{15}$ $= 40 + \frac{538\frac{8}{9}}{15}$ $= 75\frac{25}{27}$
	$=$	$= \beta_{15,75\frac{25}{27}}$
D^{f_1}		$\beta_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1} = b^{f_1}$ $15 \cdot \left[t - 75\frac{25}{27}\right]^+ = 25$ $t = 77\frac{16}{27}$
B^{f_1}		$\alpha^{f_1}\left(T_{\langle s_2,s_3\rangle}^{\text{l.o.}f_1}\right) = 5 \cdot 75\frac{25}{27} + 25$ $= 404\frac{17}{27}$

Flow f_2

Total Flow Analysis

Arrival Bounds

$(s_1, \{f_0\}, \emptyset) =: \alpha_{s_1}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0}$			
$\alpha_{s_0}^{x(f_0)}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$		$= \beta_{s_0} = \beta_{20,20}$	
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0} = \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$	$r_{s_1}^{f_0}$	$= 5$	
	$b_{s_1}^{f_0}$	$= \alpha^{f_0}\left(T_{s_0}^{\text{l.o.}}\right) = 5 \cdot 20 + 25 = 125$	
	$=$	$= \gamma_{5,125}$	

$(s_1, \{f_2\}, \emptyset) =: \alpha_{s_1}^{f_2}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_1}^{f_2} = \alpha^{f_2} \oslash \beta_{s_2}^{\text{l.o.}f_2}$			
$\alpha_{s_2}^{x(f_2)}$		$\alpha^{f_1} = \gamma_{5,25}$	
$\beta_{s_2}^{\text{l.o.}f_2} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_2)} = \beta_{R_{s_2}^{\text{l.o.}f_2}, T_{s_2}^{\text{l.o.}f_2}}$	$R_{s_2}^{\text{l.o.}f_2}$	$\left[R_{s_2} - r_{s_2}^{x(f_2)}\right]^+ = 20 - 5 = 15$	
	$T_{s_2}^{\text{l.o.}f_2}$	$\beta_{s_2} = b_{s_2}^{x(f_2)}$; $20 \cdot [t-20]^+ = 25$; $t = 21\frac{1}{4}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_2)}$; $20 \cdot [t-20]^+ = 5 \cdot t + 25$; $t = 28\frac{1}{3}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
$\alpha_{s_1}^{f_2} = \alpha^{f_2} \oslash \beta_{s_2}^{\text{l.o.}f_2} = \gamma_{r_{s_1}^{f_2}, b_{s_1}^{f_2}}$	$r_{s_1}^{f_2}$	$= 5$	
	$b_{s_1}^{f_2}$	$= \alpha^{f_2}\left(T_{s_2}^{\text{l.o.}f_2}\right) = 5 \cdot 21\frac{1}{4} + 25 = 131\frac{1}{4}$	$= \alpha^{f_2}\left(T_{s_2}^{\text{l.o.}f_2}\right) = 5 \cdot 28\frac{1}{3} + 25 = 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$

Analysis

<table>
<tr><th colspan="2">TFA</th><th>FIFO Multiplexing</th><th>Arbitrary Multiplexing</th></tr>
<tr><td rowspan="3">s_2</td><td>α_{s_2}</td><td colspan="2">$\begin{aligned} &= \alpha^{f_1} + \alpha^{f_2} \\ &= \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{10,50} \end{aligned}$</td></tr>
<tr><td>$D^{f_2}_{s_2}$</td><td>$\begin{aligned} \beta_{s_2} &= b_{s_2} \\ 20 \cdot [t-20]^+ &= 50 \\ t &= 22\frac{1}{2} \end{aligned}$</td><td>$\begin{aligned} \beta_{s_2} &= \alpha_{s_2} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 50 \\ t &= 45 \end{aligned}$</td></tr>
<tr><td>$B^{f_2}_{s_2}$</td><td colspan="2">$\begin{aligned} \alpha_{s_2}\left(T_{s_2}\right) &= 20 \cdot 10 + 50 \\ &= 250 \end{aligned}$</td></tr>
<tr><td rowspan="3">s_1</td><td>α_{s_3}</td><td>$\begin{aligned} &= \alpha^{f_0}_{s_3} + \alpha^{f_1}_{s_3} \\ &= \gamma_{5,125} + \gamma_{5,131\frac{1}{4}} \\ &= \gamma_{10,256\frac{1}{4}} \end{aligned}$</td><td>$\begin{aligned} &= \alpha^{f_0}_{s_3} + \alpha^{f_1}_{s_3} \\ &= \gamma_{5,125} + \gamma_{5,166\frac{2}{3}} \\ &= \gamma_{10,291\frac{2}{3}} \end{aligned}$</td></tr>
<tr><td>$D^{f_2}_{s_1}$</td><td>$\begin{aligned} \beta_{s_1} &= b_{s_1} \\ 20 \cdot [t-20]^+ &= 256\frac{1}{4} \\ t &= 32\frac{13}{16} \end{aligned}$</td><td>$\begin{aligned} \beta_{s_1} &= \alpha_{s_1} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 291\frac{2}{3} \\ t &= 69\frac{1}{6} \end{aligned}$</td></tr>
<tr><td>$B^{f_2}_{s_1}$</td><td>$\begin{aligned} \alpha_{s_1}\left(T_{s_1}\right) &= 10 \cdot 20 + 256\frac{1}{4} \\ &= 456\frac{1}{4} \end{aligned}$</td><td>$\begin{aligned} \alpha_{s_1}\left(T_{s_1}\right) &= 10 \cdot 20 + 291\frac{2}{3} \\ &= 491\frac{2}{3} \end{aligned}$</td></tr>
<tr><td colspan="2">D^{f_2}</td><td>$\begin{aligned} &= \sum_{i=1}^{2} D^{f_2}_{s_i} \\ &= 55\frac{5}{16} \end{aligned}$</td><td>$\begin{aligned} &= \sum_{i=1}^{2} D^{f_2}_{s_i} \\ &= 114\frac{1}{6} \end{aligned}$</td></tr>
<tr><td colspan="2">B^{f_2}</td><td>$\begin{aligned} &= \max_{i=\{1,2\}} B^{f_2}_{s_i} \\ &= 456\frac{1}{4} \end{aligned}$</td><td>$\begin{aligned} &= \max_{i=\{1,2\}} B^{f_2}_{s_i} \\ &= 491\frac{2}{3} \end{aligned}$</td></tr>
</table>

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

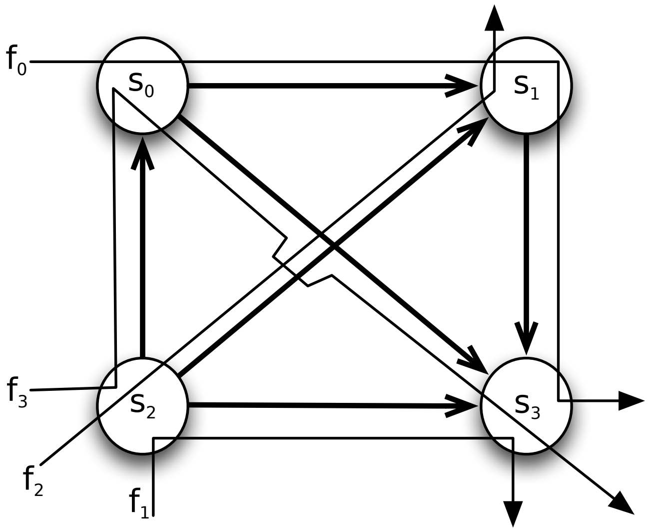
$(s_1, \{f_0\}, f_2) =: \alpha_{s_1}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0}$			
$\alpha_{s_0}^{x(f_0)}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$		$= \beta_{s_0} = \beta_{20,20}$	
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0} = \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$	$r_{s_1}^{f_0}$	$= 5$	
	$b_{s_1}^{f_0}$	$= \alpha^{f_0}\left(T_{s_0}^{\text{l.o.}}\right)$ $= 5 \cdot 20 + 25$ $= 125$	
	$=$	$= \gamma_{5,125}$	

Analyses

SFA			FIFO Multiplexing	Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_2)}$		$= \alpha^{f_1} = \gamma_{5,25}$	
	$\beta_{s_2}^{\text{l.o.}f_2} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)}$		$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s_1	$\alpha_{s_1}^{x(f_2)}$		$= \alpha_{s_1}^{f_0} = \gamma_{5,125}$	
	$\beta_{s_1}^{\text{l.o.}f_2} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_2)}$ $= \beta_{R_{s_1}^{\text{l.o.}f_2}, T_{s_1}^{\text{l.o.}f_2}}$	$R_{s_1}^{\text{l.o.}f_2}$	$\left[R_{s_1} - r_{s_1}^{x(f_2)}\right]^+ = 20 - 5$ $= 15$	
		$T_{s_1}^{\text{l.o.}f_2}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t-20]^+ = 125$ $t = 26\frac{1}{4}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_2)}$ $20 \cdot [t-20]^+ = 5 \cdot t + 125$ $t = 35$
		$=$	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$
$\beta_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2} = \beta_{R_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2}, T_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2}}$			$= \bigotimes_{i=1}^{2} \beta_{s_i}^{\text{l.o.}f_2}$ $= \beta_{15,47\frac{1}{2}}$	$= \bigotimes_{i=1}^{2} \beta_{s_i}^{\text{l.o.}f_2}$ $= \beta_{15,63\frac{1}{3}}$
D^{f_2}			$\beta_{\text{e2e}}^{\text{l.o.}f_2} = b^{f_2}$ $15 \cdot \left[t - 47\frac{1}{2}\right]^+ = 25$ $t = 49\frac{1}{6}$	$\beta_{\text{e2e}}^{\text{l.o.}f_2} = b^{f_2}$ $15 \cdot \left[t - 63\frac{1}{3}\right]^+ = 25$ $t = 65$
B^{f_2}			$\alpha^{f_2}\left(T_{\text{e2e}}^{\text{l.o.}f_2}\right) = 5 \cdot 47\frac{1}{2} + 25$ $= 262\frac{1}{2}$	$\alpha^{f_2}\left(T_{\text{e2e}}^{\text{l.o.}f_2}\right) = 5 \cdot 63\frac{1}{3} + 25$ $= 341\frac{2}{3}$

PMOO		Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_2)}$	$= \alpha^{f_1} = \gamma_{5,25}$
	$\alpha_{s_2}^{\bar{x}(f_2)}$	
s_1	$\alpha_{s_1}^{x(f_2)}$	$= \alpha_{s_1}^{f_0} = \gamma_{5,125}$
	$\alpha_{s_1}^{\bar{x}(f_2)}$	
$\beta_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2} = \beta_{R_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2},T_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2}}$	$R_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2}$	$\begin{aligned} &= \bigwedge_{i\in\{2,1\}} \left(R_{s_i} - r_{s_i}^{x(f_2)}\right) \\ &= (20-5) \wedge (20-5) \\ &= 15 \end{aligned}$
	$T_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2}$	$\begin{aligned} &= \sum_{i\in\{2,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_2)} + r_{s_i}^{x(f_2)} \cdot T_{s_i}}{R_{\text{e2e}}^{\text{l.o.}f_2}}\right) \\ &= 20 + \frac{25+5\cdot 20}{15} + 20 + \frac{125+5\cdot 20}{15} \\ &= 40 + \frac{350}{15} \\ &= 63\frac{1}{3} \end{aligned}$
	$=$	$= \beta_{15,63\frac{1}{3}}$
D^{f_2}		$\begin{aligned} \beta_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2} &= b^{f_2} \\ 15 \cdot \left[t - 63\frac{1}{3}\right]^{+} &= 25 \\ t &= 65 \end{aligned}$
B^{f_2}		$\begin{aligned} \alpha^{f_2}\left(T_{\langle s_2,s_1\rangle}^{\text{l.o.}f_2}\right) &= 5 \cdot 63\frac{1}{3} + 25 \\ &= 341\frac{2}{3} \end{aligned}$

FeedForward_1SC_4Flows_1AC_4Paths



$\mathbb{S} = \{s_0, s_1, s_2, s_3\}$ with

$$\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{s_3} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20},\ i \in \{0, 1, 2, 3\}$$

$\mathbb{F} = \{f_0, f_1, f_2, f_3\}$ with

$$\alpha^{f_n} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25},\ n \in \{0, 1, 2, 3\}$$

Arrival Bounds

$(s_0,\{f_3\},\emptyset) =: \alpha_{s_0}^{f_3}$ $(s_1,\{f_2\},\emptyset) =: \alpha_{s_1}^{f_2}$ $(s_3,\{f_1\},\emptyset) =: \alpha_{s_3}^{f_1}$ $=: \alpha_{s_i}^{f_n}$ with $(n,i)\in\{(3,0),(2,1),(1,3)\}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_2}^{\text{l.o.}f_n}$			
$\alpha_{s_2}^{x(f_n)}$		$=\gamma_{5,25}+\gamma_{5,25}=\gamma_{10,50}$	
$\beta_{s_2}^{\text{l.o.}f_n} = \beta_{s_2}\ominus\alpha^{x(f_n)}$ $= \beta_{R_{s_2}^{\text{l.o.}f_n},T_{s_2}^{\text{l.o.}f_n}}$	$R_{s_2}^{\text{l.o.}f_n}$	$\left[R_{s_2}-r_{s_2}^{x(f_n)}\right]^+ = 20-5$ $= 15$	
	$T_{s_2}^{\text{l.o.}f_n}$	$\beta_{s_2} = b_{s_2}^{x(f_n)}$ $20\cdot[t-20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_n)}$ $20\cdot[t-20]^+ = 10\cdot t+50$ $t = 45$
	$=$	$=\beta_{10,22\frac{1}{2}}$	$=\beta_{10,45}$
$\alpha_{s_i}^{f_n} = \alpha^{f_n}\oslash\beta_{s_2}^{\text{l.o.}f_n}$ $= \gamma_{r_{s_i}^{f_n},b_{s_i}^{f_n}}$	$r_{s_i}^{f_n}$	$=5$	
	$b_{s_i}^{f_n}$	$\alpha^{f_n}\left(T_{s_2}^{\text{l.o.}f_n}\right)=5\cdot 22\frac{1}{2}+25=137\frac{1}{2}$	$\alpha^{f_n}\left(T_{s_2}^{\text{l.o.}f_n}\right)=5\cdot 45+25=250$
	$=$	$=\gamma_{5,137\frac{1}{2}}$	$=\gamma_{5,250}$

$(s_3,\{f_0\},\emptyset) = \alpha_{s_1}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_1}^{f_0} = \alpha^{f_0}\oslash\beta_{s_0}^{\text{l.o.}f_0}$			
$\alpha_{s_0}^{f_0}$		$=\alpha^{f_0}=\gamma_{5,25}$	
$\alpha_{s_0}^{x(f_0)}$		$=\alpha_{s_0}^{f_3}=\gamma_{5,137\frac{1}{2}}$	$=\alpha_{s_0}^{f_3}=\gamma_{5,250}$
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0}\ominus\alpha_{s_0}^{x(f_0)}$ $= \beta_{R_{s_0}^{\text{l.o.}f_0},T_{s_0}^{\text{l.o.}f_0}}$	$R_{s_0}^{\text{l.o.}f_0}$	$\left[R_{s_0}-r_{s_0}^{x(f_0)}\right]^+ = 20-5$ $= 15$	
	$T_{s_0}^{\text{l.o.}f_0}$	$\beta_{s_0} = b_{s_0}^{x(f_0)}$ $20\cdot[t-20]^+ = 137\frac{1}{2}$ $t = 26\frac{7}{8}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$ $20\cdot[t-20]^+ = 5\cdot t+250$ $t = 43\frac{1}{3}$
	$=$	$=\beta_{15,26\frac{7}{8}}$	$=\beta_{15,43\frac{1}{3}}$
$\alpha_{s_1}^{f_0} = \alpha^{f_0}\oslash\beta_{s_0}^{\text{l.o.}f_0}$ $= \gamma_{r_{s_1}^{f_0},b_{s_1}^{f_0}}$	$r_{s_1}^{f_0}$	$=5$	
	$b_{s_1}^{f_0}$	$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_0}\right)=5\cdot 26\frac{7}{8}+25=159\frac{3}{8}$	$\alpha^{f_0}\left(T_{s_0}^{\text{l.o.}f_0}\right)=5\cdot 43\frac{1}{3}+25=241\frac{2}{3}$
	$=$	$=\gamma_{5,159\frac{3}{8}}$	$=\gamma_{5,241\frac{2}{3}}$

$(s_3,\{f_0\},\emptyset) = \alpha_{s_3}^{f_0}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_3}^{f_0} = \alpha^{f_0}\oslash\left(\beta_{s_0}^{\text{l.o.}f_0}\otimes\beta_{s_1}^{\text{l.o.}f_0}\right)$ (reuse of previous result) $= \alpha^{f_0}\oslash\beta_{s_0}^{\text{l.o.}f_0}\oslash\beta_{s_1}^{\text{l.o.}f_0}$ $= \alpha_{s_1}^{f_0}\oslash\beta_{s_1}^{\text{l.o.}f_0}$			
$\alpha_{s_1}^{f_0}$		$=\gamma_{5,159\frac{3}{8}}$	$=\gamma_{5,241\frac{2}{3}}$
$\alpha_{s_1}^{x(f_0)}$		$=\alpha_{s_1}^{f_2}=\gamma_{5,137\frac{1}{2}}$	$=\alpha_{s_1}^{f_2}=\gamma_{5,250}$
$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1}\otimes\alpha_{s_1}^{x(f_0)}$ $= \beta_{R_{s_1}^{\text{l.o.}f_0},T_{s_1}^{\text{l.o.}f_0}}$	$R_{s_1}^{\text{l.o.}f_0}$	$\left[R_{s_1}-r_{s_1}^{x(f_0)}\right]^+ = 20-5$ $= 15$	
	$T_{s_1}^{\text{l.o.}f_0}$	$\beta_{s_1} = b_{s_1}^{x(f_0)}$ $20\cdot[t-20]^+ = 137\frac{1}{2}$ $t = 26\frac{7}{8}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$ $20\cdot[t-20]^+ = 5\cdot t+250$ $t = 43\frac{1}{3}$
	$=$	$=\beta_{15,26\frac{7}{8}}$	$=\beta_{15,43\frac{1}{3}}$
$\alpha_{s_3}^{f_0} = \alpha_{s_1}^{f_0}\oslash\beta_{s_1}^{\text{l.o.}f_0}$ $= \gamma_{r_{s_3}^{f_0},b_{s_3}^{f_0}}$	$r_{s_3}^{f_0}$	$=5$	
	$b_{s_3}^{f_0}$	$\alpha_{s_1}^{f_0}\left(T_{s_1}^{\text{l.o.}f_0}\right)=5\cdot 26\frac{7}{8}+159\frac{3}{8}=293\frac{3}{4}$	$\alpha_{s_1}^{f_0}\left(T_{s_1}^{\text{l.o.}f_0}\right)=5\cdot 43\frac{1}{3}+241\frac{2}{3}=458\frac{1}{3}$
	$=$	$=\gamma_{5,293\frac{3}{4}}$	$=\gamma_{5,458\frac{1}{3}}$

$(s_3, \{f_3\}, \emptyset) = \alpha^{f_3}_{s_3}$		FIFO Multiplexing	Arbitrary Multiplexing
`PbooArrivalBound_Concatenation.java`			
$\alpha^{f_3}_{s_3} = \alpha^{f_3} \oslash \left(\beta^{\text{l.o.}f_3}_{s_2} \otimes \beta^{\text{l.o.}f_3}_{s_0}\right)$ (reuse of previous result) $= \alpha^{f_3} \oslash \beta^{\text{l.o.}f_3}_{s_2} \oslash \beta^{\text{l.o.}f_3}_{s_0}$ $= \alpha^{f_3}_{s_0} \oslash \beta^{\text{l.o.}f_3}_{s_0}$			
$\alpha^{x(f_3)}_{s_0}$		$= \alpha^{f_0} = \gamma_{5,25}$	
$\beta^{\text{l.o.}f_3}_{s_0} = \beta_{s_0} \ominus \alpha^{x(f_3)}_{s_0}$ $= \beta_{R^{\text{l.o.}f_3}_{s_0}, T^{\text{l.o.}f_3}_{s_0}}$	$R^{\text{l.o.}f_3}_{s_0}$	$\left[R_{s_0} - r^{x(f_3)}_{s_0}\right]^+ = 20 - 5$ $= 15$	
	$T^{\text{l.o.}f_3}_{s_0}$	$\beta_{s_0} = b^{f_0}$ $20 \cdot [t-20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_0} = \alpha^{f_0}$ $20 \cdot [t-20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
$\alpha^{f_3}_{s_0}$		$= \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,250}$
$\alpha^{f_3}_{s_3} = \alpha^{f_3}_{s_0} \oslash \beta^{\text{l.o.}f_3}_{s_0}$ $= \gamma_{r^{f_3}_{s_3}, b^{f_3}_{s_3}}$	$r^{f_3}_{s_3}$	$= 5$	
	$b^{f_3}_{s_3}$	$= \alpha^{f_3}_{s_0}\left(T^{\text{l.o.}f_3}_{s_0}\right)$ $= 5 \cdot 21\frac{1}{4} + 137\frac{1}{2}$ $= 243\frac{3}{4}$	$= \alpha^{f_3}_{s_0}\left(T^{\text{l.o.}f_3}_{s_0}\right)$ $= 5 \cdot 28\frac{1}{3} + 250$ $= 391\frac{2}{3}$
	$=$	$= \gamma_{5,243\frac{3}{4}}$	$= \gamma_{5,391\frac{2}{3}}$

`PmooArrivalBound.java`			
$\alpha^{f_3}_{s_3} = \alpha^{f_3} \oslash \beta^{\text{l.o.}f_3}_{\langle s_2, s_0\rangle}$			
s_2	$\alpha^{x(f_3)}_{s_2}$		$= \alpha^{f_1} + \alpha^{f_2}$ $= \gamma_{5,25} + \gamma_{5,25}$
	$\alpha^{\bar{x}(f_3)}_{s_2}$		
s_0	$\alpha^{x(f_3)}_{s_0}$		$= \gamma_{10,50}$ $= \alpha^{f_0} = \gamma_{5,25}$
	$\alpha^{\bar{x}(f_3)}_{s_0}$		
$\beta^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle} = \beta_{R^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle}, T^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle}}$	$R^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle}$		$= \bigwedge_{i\in\{2,0\}} \left(R_{s_i} - r^{x(f_3)}_{s_i}\right)$ $= (20-10) \wedge (20-5)$ $= 10$
	$T^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle}$		$= \sum_{i\in\{2,0\}} \left(T_{s_i} + \frac{b^{\bar{x}(f_3)}_{s_i} + r^{x(f_3)}_{s_i} \cdot T_{s_i}}{R^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle}}\right)$ $= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{25 + 5 \cdot 20}{10}$ $= 77\frac{1}{2}$
	$=$		$= \beta_{10,77\frac{1}{2}}$
$\alpha^{f_3}_{s_3} = \alpha^{f_3} \oslash \beta^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle}$	$r^{f_3}_{s_3}$		$= 5$
	$b^{f_3}_{s_3}$		$= \alpha^{f_3}(T^{\text{l.o.}f_3}_{\langle s_2,s_0\rangle})$ $= 5 \cdot 77\frac{1}{2} + 25$ $= 412\frac{1}{2}$
	$=$		$= \gamma_{5,412\frac{1}{2}}$

Flow f_0

Total Flow Analysis

Analysis

<table>
<tr><td rowspan="2" colspan="2">TFA</td><td>FIFO Multiplexing</td><td colspan="2">Arbitrary Multiplexing</td></tr>
<tr><td colspan="2"><code>PbooArrivalBound_Concatenation.java</code></td><td><code>PmooArrivalBound.java</code></td></tr>
<tr><td rowspan="3">s_0</td><td>α_{s_0}</td><td>$\begin{aligned} &= \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,137\frac{1}{2}} \\ &= \gamma_{10,162\frac{1}{2}} \end{aligned}$</td><td colspan="2">$\begin{aligned} &= \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,250} \\ &= \gamma_{10,275} \end{aligned}$</td></tr>
<tr><td>$D_{s_0}^{f_0}$</td><td>$\begin{aligned} \beta_{s_0} &= b_{s_0} \\ 20 \cdot [t-20]^{+} &= 162\frac{1}{2} \\ t &= 28\frac{1}{8} \end{aligned}$</td><td colspan="2">$\begin{aligned} \beta_{s_0} &= \alpha_{s_0} \\ 20 \cdot [t-20]^{+} &= 10 \cdot t + 275 \\ t &= 67\frac{1}{2} \end{aligned}$</td></tr>
<tr><td>$B_{s_0}^{f_0}$</td><td>$\begin{aligned} \alpha_{s_0}\left(T_{s_0}\right) &= 10 \cdot 20 + 162\frac{1}{2} \\ &= 262\frac{1}{2} \end{aligned}$</td><td colspan="2">$\begin{aligned} \alpha_{s_0}\left(T_{s_0}\right) &= 10 \cdot 20 + 275 \\ &= 475 \end{aligned}$</td></tr>
<tr><td rowspan="3">s_1</td><td>α_{s_1}</td><td>$\begin{aligned} &= \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_2} \\ &= \gamma_{5,159\frac{3}{8}} + \gamma_{5,137\frac{1}{2}} \\ &= \gamma_{10,296\frac{7}{8}} \end{aligned}$</td><td colspan="2">$\begin{aligned} &= \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_2} \\ &= \gamma_{5,241\frac{2}{3}} + \gamma_{5,250} \\ &= \gamma_{10,491\frac{2}{3}} \end{aligned}$</td></tr>
<tr><td>$D_{s_1}^{f_0}$</td><td>$\begin{aligned} \beta_{s_1} &= b_{s_1} \\ 20 \cdot [t-20]^{+} &= 296\frac{7}{8} \\ t &= 34\frac{27}{32} \end{aligned}$</td><td colspan="2">$\begin{aligned} \beta_{s_1} &= \alpha_{s_1} \\ 20 \cdot [t-20]^{+} &= 10 \cdot t + 491\frac{2}{3} \\ t &= 89\frac{1}{6} \end{aligned}$</td></tr>
<tr><td>$B_{s_1}^{f_0}$</td><td>$\begin{aligned} \alpha_{s_1}\left(T_{s_1}\right) &= 10 \cdot 20 + 296\frac{7}{8} \\ &= 496\frac{7}{8} \end{aligned}$</td><td colspan="2">$\begin{aligned} \alpha_{s_1}\left(T_{s_1}\right) &= 10 \cdot 20 + 491\frac{2}{3} \\ &= 691\frac{2}{3} \end{aligned}$</td></tr>
<tr><td rowspan="3">s_3</td><td>α_{s_3}</td><td>$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}} \\ &= \gamma_{15,675} \end{aligned}$</td><td>$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,391\frac{2}{3}} \\ &= \gamma_{15,1100} \end{aligned}$</td><td>$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,412\frac{1}{2}} \\ &= \gamma_{15,1120\frac{5}{6}} \end{aligned}$</td></tr>
<tr><td>$D_{s_3}^{f_0}$</td><td>$\begin{aligned} \beta_{s_3} &= b_{s_3} \\ 20 \cdot [t-20]^{+} &= 675 \\ t &= 53\frac{3}{4} \end{aligned}$</td><td>$\begin{aligned} \beta_{s_3} &= \alpha_{s_3} \\ 20 \cdot [t-20]^{+} &= 15 \cdot t + 1100 \\ t &= 300 \end{aligned}$</td><td>$\begin{aligned} \beta_{s_3} &= \alpha_{s_3} \\ 20 \cdot [t-20]^{+} &= 15 \cdot t + 1120\frac{5}{6} \\ t &= 304\frac{1}{6} \end{aligned}$</td></tr>
<tr><td>$B_{s_3}^{f_0}$</td><td>$\begin{aligned} \alpha_{s_3}\left(T_{s_3}\right) &= 15 \cdot 20 + 675 \\ &= 975 \end{aligned}$</td><td>$\begin{aligned} \alpha_{s_3}\left(T_{s_3}\right) &= 15 \cdot 20 + 1100 \\ &= 1400 \end{aligned}$</td><td>$\begin{aligned} \alpha_{s_3}\left(T_{s_3}\right) &= 15 \cdot 20 + 1120\frac{5}{6} \\ &= 1420\frac{5}{6} \end{aligned}$</td></tr>
<tr><td colspan="2">D^{f_0}</td><td>$D_{s_0}^{f_0} + D_{s_1}^{f_0} + D_{s_3}^{f_0} = 116\frac{23}{32}$</td><td>$D_{s_0}^{f_0} + D_{s_1}^{f_0} + D_{s_3}^{f_0} = 456\frac{2}{3}$</td><td>$D_{s_0}^{f_0} + D_{s_1}^{f_0} + D_{s_3}^{f_0} = 460\frac{5}{6}$</td></tr>
<tr><td colspan="2">B^{f_0}</td><td>$\max\left\{B_{s_0}^{f_0}, B_{s_1}^{f_0}, B_{s_3}^{f_0}\right\} = 975$</td><td>$\max\left\{B_{s_0}^{f_0}, B_{s_1}^{f_0}, B_{s_3}^{f_0}\right\} = 1400$</td><td>$\max\left\{B_{s_0}^{f_0}, B_{s_1}^{f_0}, B_{s_3}^{f_0}\right\} = 1420\frac{5}{6}$</td></tr>
</table>

Separate Flow Analysis and PMOO Analysis

Analyses

`PbooArrivalBound_Concatenation.java`

Server	SFA		FIFO Multiplexing	Arbitrary Multiplexing
s_0	$\alpha^{x(f_0)}_{s_0}$		$= \alpha^{f_3}_{s_0} = \gamma_{5,137\frac{1}{2}}$	$= \alpha^{f_3}_{s_0} = \gamma_{5,250}$
	$\begin{aligned} \beta^{\text{l.o.}f_0}_{s_0} &= \beta_{s_0} \ominus \alpha^{x(f_3)}_{s_0} \\ &= \beta_{R^{\text{l.o.}f_0}_{s_0}, T^{\text{l.o.}f_0}_{s_0}} \end{aligned}$	$R^{\text{l.o.}f_0}_{s_0}$	$\left[R_{s_0} - r^{x(f_0)}_{s_0}\right]^+ = 15$	
		$T^{\text{l.o.}f_0}_{s_0}$	$\begin{aligned} \beta_{s_0} &= b^{x(f_0)}_{s_0} \\ 20 \cdot [t-20]^+ &= 137\frac{1}{2} \\ t &= 26\frac{7}{8} \end{aligned}$	$\begin{aligned} \beta_{s_0} &= \alpha^{x(f_0)}_{s_0} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 250 \\ t &= 43\frac{1}{3} \end{aligned}$
		$=$	$= \beta_{15,26\frac{7}{8}}$	$= \beta_{15,43\frac{1}{3}}$
s_1	$\alpha^{x(f_0)}_{s_1}$		$= \alpha^{f_1}_{s_1} = \gamma_{5,137\frac{1}{2}}$	$= \alpha^{f_1}_{s_1} = \gamma_{5,250}$
	$\begin{aligned} \beta^{\text{l.o.}f_0}_{s_1} &= \beta_{s_1} \ominus \alpha^{x(f_0)}_{s_1} \\ &= \beta_{R^{\text{l.o.}f_0}_{s_1}, T^{\text{l.o.}f_0}_{s_1}} \end{aligned}$	$R^{\text{l.o.}f_0}_{s_1}$	$\left[R_{s_1} - r^{x(f_0)}_{s_1}\right]^+ = 15$	
		$T^{\text{l.o.}f_0}_{s_1}$	$\begin{aligned} \beta_{s_1} &= b^{x(f_0)}_{s_1} \\ 20 \cdot [t-20]^+ &= 137\frac{1}{2} \\ t &= 26\frac{7}{8} \end{aligned}$	$\begin{aligned} \beta_{s_1} &= \alpha^{x(f_0)}_{s_1} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 250 \\ t &= 43\frac{1}{3} \end{aligned}$
		$=$	$= \beta_{15,26\frac{7}{8}}$	$= \beta_{15,43\frac{1}{3}}$
s_3	$\alpha^{x(f_0)}_{s_3}$		$\begin{aligned} &= \alpha^{f_2}_{s_3} + \alpha^{f_3}_{s_3} \\ &= \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}} \\ &= \gamma_{10,381\frac{1}{4}} \end{aligned}$	$\begin{aligned} &= \alpha^{f_2}_{s_3} + \alpha^{f_3}_{s_3} \\ &= \gamma_{5,250} + \gamma_{5,391\frac{2}{3}} \\ &= \gamma_{10,641\frac{2}{3}} \end{aligned}$
	$\begin{aligned} \beta^{\text{l.o.}f_0}_{s_3} &= \beta_{s_3} \ominus \alpha^{x(f_0)}_{s_3} \\ &= \beta_{R^{\text{l.o.}f_0}_{s_3}, T^{\text{l.o.}f_0}_{s_3}} \end{aligned}$	$R^{\text{l.o.}f_0}_{s_3}$	$\left[R_{s_3} - r^{x(f_0)}_{s_3}\right]^+ = 10$	
		$T^{\text{l.o.}f_0}_{s_3}$	$\begin{aligned} \beta_{s_3} &= b^{x(f_0)}_{s_3} \\ 20 \cdot [t-20]^+ &= 381\frac{1}{4} \\ t &= 39\frac{1}{16} \end{aligned}$	$\begin{aligned} \beta_{s_3} &= \alpha^{x(f_0)}_{s_3} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 641\frac{2}{3} \\ t &= 104\frac{1}{6} \end{aligned}$
		$=$	$= \beta_{10,39\frac{1}{16}}$	$= \beta_{10,104\frac{1}{6}}$
$\beta^{\text{l.o.}f_0}_{\text{e2e}} = \beta_{R^{\text{l.o.}f_0}_{\text{e2e}}, T^{\text{l.o.}f_0}_{\text{e2e}}}$			$\bigotimes_{i=\{0,1,3\}} \beta^{\text{l.o.}f_0}_{s_i} = \beta_{10,92\frac{13}{16}}$	$\bigotimes_{i=\{0,1,3\}} \beta^{\text{l.o.}f_0}_{s_i} = \beta_{10,190\frac{5}{6}}$
D^{f_0}			$\begin{aligned} \beta^{\text{l.o.}f_0}_{\text{e2e}} &= b^{f_0} \\ 10 \cdot \left[t - 92\frac{13}{16}\right]^+ &= 25 \\ t &= 95\frac{5}{16} \end{aligned}$	$\begin{aligned} \beta^{\text{l.o.}f_0}_{\text{e2e}} &= b^{f_0} \\ 10 \cdot \left[t - 190\frac{5}{6}\right]^+ &= 25 \\ t &= 193\frac{1}{3} \end{aligned}$
B^{f_0}			$\begin{aligned} \alpha^{f_0}\left(T^{\text{l.o.}f_0}_{\text{e2e}}\right) &= 5 \cdot 92\frac{13}{16} + 25 \\ &= 489\frac{1}{16} \end{aligned}$	$\begin{aligned} \alpha^{f_0}\left(T^{\text{l.o.}f_0}_{\text{e2e}}\right) &= 5 \cdot 190\frac{5}{6} + 25 \\ &= 979\frac{1}{6} \end{aligned}$

`PmooArrivalBound.java`

	SFA		Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_0)}$		$= \alpha_{s_0}^{f_3} = \gamma_{5,250}$
	$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_3)}$ $= \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$	$R_{s_0}^{\text{l.o.}f_0}$	$\left[R_{s_0} - r_{s_0}^{x(f_0)}\right]^+ = 15$
		$T_{s_0}^{\text{l.o.}f_0}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$ $20 \cdot [t-20]^+ = 5 \cdot t + 250$ $t = 43\frac{1}{3}$
		$=$	$= \beta_{15,43\frac{1}{3}}$
s_1	$\alpha_{s_1}^{x(f_0)} = \alpha_{s_1}^{f_1}$		$= \gamma_{5,250}$
	$\beta_{s_1}^{\text{l.o.}f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$ $= \beta_{R_{s_1}^{\text{l.o.}f_0}, T_{s_1}^{\text{l.o.}f_0}}$	$R_{s_1}^{\text{l.o.}f_0}$	$\left[R_{s_1} - r_{s_1}^{x(f_0)}\right]^+ = 15$
		$T_{s_1}^{\text{l.o.}f_0}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$ $20 \cdot [t-20]^+ = 5 \cdot t + 250$ $t = 43\frac{1}{3}$
		$=$	$= \beta_{15,43\frac{1}{3}}$
s_3	$\alpha_{s_3}^{x(f_0)} = \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3}$		$= \gamma_{5,250} + \gamma_{5,412\frac{1}{2}} = \gamma_{10,662\frac{1}{2}}$
	$\beta_{s_3}^{\text{l.o.}f_0} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_0)}$ $= \beta_{R_{s_3}^{\text{l.o.}f_0}, T_{s_3}^{\text{l.o.}f_0}}$	$R_{s_3}^{\text{l.o.}f_0}$	$\left[R_{s_3} - r_{s_3}^{x(f_0)}\right]^+ = 10$
		$T_{s_3}^{\text{l.o.}f_0}$	$\beta_{s_3} = \alpha_{s_3}^{x(f_0)}$ $20 \cdot [t-20]^+ = 10 \cdot t + 662\frac{1}{2}$ $t = 106\frac{1}{4}$
		$=$	$= \beta_{10,106\frac{1}{4}}$
$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}}$			$\bigotimes_{i=\{0,1,3\}} \beta_{s_i}^{\text{l.o.}f_0} = \beta_{10,192\frac{11}{12}}$
D^{f_0}			$\beta_{\text{e2e}}^{\text{l.o.}f_0} = b^{f_0}$ $10 \cdot \left[t - 192\frac{11}{12}\right]^+ = 25$ $t = 195\frac{5}{12}$
B^{f_0}			$\alpha^{f_0}\left(T_{\text{e2e}}^{\text{l.o.}f_0}\right) = 5 \cdot 192\frac{11}{12} + 25$ $= 989\frac{7}{12}$

`PbooArrivalBound_Concatenation.java`

PMOO		Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_0)}$	$=\alpha_{s_0}^{f_3}=\gamma_{5,250}$
	$\alpha_{s_0}^{\bar{x}(f_0)}$	
s_1	$\alpha_{s_1}^{x(f_0)}$	$=\alpha_{s_1}^{f_2}=\gamma_{5,250}$
	$\alpha_{s_1}^{\bar{x}(f_0)}$	
s_3	$\alpha_{s_3}^{x(f_0)}$	$\begin{aligned} &= \alpha_{s_3}^{f_2}+\alpha_{s_3}^{f_3} \\ &= \gamma_{5,250}+\gamma_{5,391\frac{2}{3}} \\ &= \gamma_{5,641\frac{2}{3}} \end{aligned}$
	$\alpha_{s_3}^{\bar{x}(f_0)}$	
$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}=\beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0},T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}$	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}$	$\begin{aligned} &= \bigwedge_{i\in\{0,1,3\}}\left(R_{s_i}-r_{s_i}^{x(f_0)}\right) \\ &= (20-5)\wedge(20-5)\wedge(20-10) \\ &= 10 \end{aligned}$
	$T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}$	$\begin{aligned} &= \sum_{i\in\{0,1,3\}}\left(T_{s_i}+\frac{b_{s_i}^{\bar{x}(f_0)}+r_{s_i}^{x(f_0)}\cdot T_{s_i}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}\right) \\ &= 20+\frac{250+5\cdot 20}{10}+20+\frac{250+5\cdot 20}{10}+20+\frac{641\frac{2}{3}+10\cdot 20}{10} \\ &= 60+\frac{1541\frac{2}{3}}{10} \\ &= 214\frac{1}{6} \end{aligned}$
	$=$	$=\beta_{10,214\frac{1}{6}}$
D^{f_0}		$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} &= b^{f_0} \\ 10\cdot\left[t-214\frac{1}{6}\right]^{+} &= 25 \\ t &= 216\frac{2}{3} \end{aligned}$
B^{f_0}		$\begin{aligned} \alpha^{f_0}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}\right) &= 5\cdot 214\frac{1}{6}+25 \\ &= 1095\frac{5}{6} \end{aligned}$

`PmooArrivalBound.java`

PMOO		Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \alpha_{s_0}^{f_3} = \gamma_{5,250}$
	$\alpha_{s_0}^{\bar{x}(f_0)}$	
s_1	$\alpha_{s_1}^{x(f_0)}$	$= \alpha_{s_1}^{f_2} = \gamma_{5,250}$
	$\alpha_{s_1}^{\bar{x}(f_0)}$	
s_3	$\alpha_{s_3}^{x(f_0)}$	$\begin{aligned} &= \alpha_{s_3}^{f_2} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,250} + \gamma_{5,412\frac{1}{2}} \\ &= \gamma_{10,662\frac{1}{2}} \end{aligned}$
	$\alpha_{s_3}^{\bar{x}(f_0)}$	
$\beta_{\text{e2e}}^{\text{l.o.}f_0} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_0}, T_{\text{e2e}}^{\text{l.o.}f_0}}$	$R_{\text{e2e}}^{\text{l.o.}f_0}$	$\begin{aligned} &= \bigwedge_{i \in \{0,1,3\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right) \\ &= (20-5) \wedge (20-5) \wedge (20-10) \\ &= 10 \end{aligned}$
	$T_{\text{e2e}}^{\text{l.o.}f_0}$	$\begin{aligned} &= \sum_{i \in \{0,1,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{\text{e2e}}^{\text{l.o.}f_0}} \right) \\ &= 20 + \frac{250 + 5 \cdot 20}{10} + 20 + \frac{250 + 5 \cdot 20}{10} + 20 + \frac{662\frac{1}{2} + 10 \cdot 20}{10} \\ &= 60 + \frac{1462\frac{1}{2}}{10} \\ &= 216\frac{1}{4} \end{aligned}$
	$=$	$= \beta_{10,216\frac{1}{4}}$
D^{f_0}		$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_0} &= b^{f_0} \\ 10 \cdot \left[t - 216\frac{1}{4} \right]^+ &= 25 \\ t &= 218\frac{3}{4} \end{aligned}$
B^{f_0}		$\begin{aligned} \alpha^{f_0}\left(T_{\text{e2e}}^{\text{l.o.}f_0} \right) &= 5 \cdot 216\frac{1}{4} + 25 \\ &= 1106\frac{1}{4} \end{aligned}$

Flow f_1

Total Flow Analysis

Analysis

`PbooArrivalBound_Concatenation.java`

<table>
<tr><th colspan="2">TFA</th><th>FIFO Multiplexing</th><th>Arbitrary Multiplexing</th></tr>
<tr><td rowspan="3">s_2</td><td>α_{s_2}</td><td colspan="2">$\begin{aligned}\alpha_{s_3}^{f_3} &= \alpha_{s_2}^{f_1}+\alpha_{s_2}^{f_2}+\alpha_{s_2}^{f_3} \\ &= \gamma_{5,25}+\gamma_{5,25}+\gamma_{5,25} \\ &= \gamma_{15,75}\end{aligned}$</td></tr>
<tr><td>$D_{s_2}^{f_1}$</td><td>$\begin{aligned}\beta_{s_2} &= b_{s_2} \\ 20\cdot[t-20]^{+} &= 75 \\ t &= 23\frac{3}{4}\end{aligned}$</td><td>$\begin{aligned}\beta_{s_2} &= \alpha_{s_2} \\ 20\cdot[t-20]^{+} &= 15\cdot t+75 \\ t &= 95\end{aligned}$</td></tr>
<tr><td>$B_{s_2}^{f_1}$</td><td colspan="2">$\begin{aligned}\alpha_{s_2}(T_{s_2}) &= 15\cdot 20+75 \\ &= 375\end{aligned}$</td></tr>
<tr><td rowspan="3">s_3</td><td>α_{s_3}</td><td>$\begin{aligned}&= \alpha_{s_3}^{f_0}+\alpha_{s_3}^{f_1}+\alpha_{s_3}^{f_3} \\ &= \gamma_{5,137\frac{1}{2}}+\gamma_{5,293\frac{3}{4}}+\gamma_{5,243\frac{3}{4}} \\ &= \gamma_{15,675}\end{aligned}$</td><td>$\begin{aligned}&= \alpha_{s_3}^{f_0}+\alpha_{s_3}^{f_1}+\alpha_{s_3}^{f_3} \\ &= \gamma_{5,250}+\gamma_{5,458\frac{1}{3}}+\gamma_{5,391\frac{2}{3}} \\ &= \gamma_{15,1100}\end{aligned}$</td></tr>
<tr><td>$D_{s_3}^{f_1}$</td><td>$\begin{aligned}\beta_{s_3} &= b_{s_3} \\ 20\cdot[t-20]^{+} &= 675 \\ t &= 53\frac{3}{4}\end{aligned}$</td><td>$\begin{aligned}\beta_{s_3} &= \alpha_{s_3} \\ 20\cdot[t-20]^{+} &= 15\cdot t+1100 \\ t &= 300\end{aligned}$</td></tr>
<tr><td>$B_{s_3}^{f_1}$</td><td>$\begin{aligned}\alpha_{s_3}(T_{s_3}) &= 15\cdot 20+675 \\ &= 975\end{aligned}$</td><td>$\begin{aligned}\alpha_{s_3}(T_{s_3}) &= 15\cdot 20+1100 \\ &= 1400\end{aligned}$</td></tr>
<tr><td colspan="2">D^{f_1}</td><td>$D_{s_2}^{f_1}+D_{s_3}^{f_1}=77\frac{1}{2}$</td><td>$D_{s_2}^{f_1}+D_{s_3}^{f_1}=395$</td></tr>
<tr><td colspan="2">B^{f_1}</td><td>$\max\left\{B_{s_2}^{f_1},B_{s_3}^{f_1}\right\}=975$</td><td>$\max\left\{B_{s_2}^{f_1},B_{s_3}^{f_1}\right\}=1400$</td></tr>
</table>

`PmooArrivalBound.java`

TFA		Arbitrary Multiplexing
s_2	α_{s_2}	$\begin{aligned} \alpha^{f_3}_{s_3} &= \alpha^{f_1}_{s_2} + \alpha^{f_2}_{s_2} + \alpha^{f_3}_{s_2} \\ &= \gamma_{5,25} + \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{15,75} \end{aligned}$
	$D^{f_1}_{s_2}$	$\begin{aligned} \beta_{s_2} &= \alpha_{s_2} \\ 20 \cdot [t-20]^{+} &= 15 \cdot t + 75 \\ t &= 95 \end{aligned}$
	$B^{f_1}_{s_2}$	$\begin{aligned} \alpha_{s_2}\left(T_{s_2}\right) &= 15 \cdot 20 + 75 \\ &= 375 \end{aligned}$
s_3	α_{s_3}	$\begin{aligned} &= \alpha^{f_0}_{s_3} + \alpha^{f_1}_{s_3} + \alpha^{f_3}_{s_3} \\ &= \gamma_{5,250} + \gamma_{5,458\frac{1}{3}} + \gamma_{5,412\frac{1}{2}} \\ &= \gamma_{15,1120\frac{5}{6}} \end{aligned}$
	$D^{f_1}_{s_3}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3} \\ 20 \cdot [t-20]^{+} &= 15 \cdot t + 1120\frac{5}{6} \\ t &= 304\frac{1}{6} \end{aligned}$
	$B^{f_1}_{s_3}$	$\begin{aligned} \alpha_{s_3}\left(T_{s_3}\right) &= 15 \cdot 20 + 1120\frac{5}{6} \\ &= 1420\frac{5}{6} \end{aligned}$
D^{f_1}		$D^{f_1}_{s_2} + D^{f_1}_{s_3} = 399\frac{1}{6}$
B^{f_1}		$\max\left\{B^{f_1}_{s_2}, B^{f_1}_{s_3}\right\} = 1420\frac{5}{6}$

Separate Flow Analysis and PMOO Analysis

Analyses

`PbooArrivalBound_Concatenation.java`

	SFA		FIFO Multiplexing	Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_1)}$		$\begin{aligned} &= \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{10,50} \end{aligned}$	
	$\beta_{s_2}^{\text{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)}$		$= \beta_{10,22\frac{1}{2}}$	$= \beta_{10,45}$
s_3	$\alpha_{s_3}^{x(f_1)}$		$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,293\frac{3}{4}} + \gamma_{5,243\frac{3}{4}} \\ &= \gamma_{10,537\frac{1}{3}} \end{aligned}$	$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,391\frac{2}{3}} \\ &= \gamma_{10,850} \end{aligned}$
	$\begin{aligned} \beta_{s_3}^{\text{l.o.}f_1} &= \beta_{s_3} \ominus \alpha_{s_3}^{x(f_1)} \\ &= \beta_{R_{s_3}^{\text{l.o.}f_1}, T_{s_3}^{\text{l.o.}f_1}} \end{aligned}$	$R_{s_3}^{\text{l.o.}f_1}$	$\left[R_{s_3} - r_{s_3}^{x(f_1)}\right]^+ = 10$	
		$T_{s_3}^{\text{l.o.}f_1}$	$\begin{aligned} \beta_{s_3} &= b_{s_3}^{x(f_1)} \\ 20 \cdot [t-20]^+ &= 537\frac{1}{2} \\ t &= 46\frac{7}{8} \end{aligned}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3}^{x(f_1)} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 850 \\ t &= 125 \end{aligned}$
		$=$	$= \beta_{10,46\frac{7}{8}}$	$= \beta_{10,125}$
$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f_1}}$			$\bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.}f_1} = \beta_{10,69\frac{3}{8}}$	$\bigotimes_{i=2}^{3} \beta_{s_i}^{\text{l.o.}f_1} = \beta_{10,170}$
D^{f_1}			$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_1} &= b^{f_1} \\ 10 \cdot \left[t - 69\frac{3}{8}\right]^+ &= 25 \\ t &= 71\frac{7}{8} \end{aligned}$	$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_1} &= b^{f_1} \\ 10 \cdot [t-170]^+ &= 25 \\ t &= 172\frac{1}{2} \end{aligned}$
B^{f_1}			$\begin{aligned} \alpha^{f_1}\left(T_{\text{e2e}}^{\text{l.o.}f_1}\right) &= 5 \cdot 69\frac{3}{8} + 25 \\ &= 371\frac{7}{8} \end{aligned}$	$\begin{aligned} \alpha^{f_1}\left(T_{\text{e2e}}^{\text{l.o.}f_1}\right) &= 5 \cdot 170 + 25 \\ &= 875 \end{aligned}$

`PmooArrivalBound.java`

	SFA		Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_1)}$		$\begin{aligned} &= \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{10,50} \end{aligned}$
	$\beta_{s_2}^{\mathbf{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)}$		$= \beta_{10,45}$
s_3	$\alpha_{s_3}^{x(f_1)}$		$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,412\frac{1}{2}} \\ &= \gamma_{10,870\frac{5}{6}} \end{aligned}$
	$\begin{aligned} \beta_{s_3}^{\mathbf{l.o.}f_1} &= \beta_{s_3} \ominus \alpha_{s_3}^{x(f_1)} \\ &= \beta_{R_{s_3}^{\mathbf{l.o.}f_1}, T_{s_3}^{\mathbf{l.o.}f_1}} \end{aligned}$	$R_{s_3}^{\mathbf{l.o.}f_1}$	$\left[R_{s_3} - r_{s_3}^{x(f_1)}\right]^+ = 10$
		$T_{s_3}^{\mathbf{l.o.}f_1}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3}^{x(f_1)} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 870\frac{5}{6} \\ t &= 127\frac{1}{12} \end{aligned}$
		$=$	$= \beta_{10,127\frac{1}{12}}$
	$\beta_{\mathbf{e2e}}^{\mathbf{l.o.}f_1} = \beta_{R_{\mathbf{e2e}}^{\mathbf{l.o.}f_1}, T_{\mathbf{e2e}}^{\mathbf{l.o.}f_1}}$		$\bigotimes_{i=2}^{3} \beta_{s_i}^{\mathbf{l.o.}f_1} = \beta_{10,172\frac{1}{12}}$
	D^{f_1}		$\begin{aligned} \beta_{\mathbf{e2e}}^{\mathbf{l.o.}f_1} &= b^{f_1} \\ 10 \cdot \left[t - 172\frac{1}{12}\right]^+ &= 25 \\ t &= 174\frac{7}{12} \end{aligned}$
	B^{f_1}		$\begin{aligned} \alpha^{f_1}\left(T_{\mathbf{e2e}}^{\mathbf{l.o.}f_1}\right) &= 5 \cdot 172\frac{1}{12} + 25 \\ &= 885\frac{5}{12} \end{aligned}$

`PbooArrivalBound_Concatenation.java`

PMOO		Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_1)}$	$\begin{aligned} &= \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{10,50} \end{aligned}$
	$\alpha_{s_2}^{\bar{x}(f_1)}$	
s_3	$\alpha_{s_3}^{x(f_1)}$	$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,391\frac{2}{3}} \\ &= \gamma_{10,850} \end{aligned}$
	$\alpha_{s_3}^{\bar{x}(f_1)}$	
$\beta_{\text{e2e}}^{\text{l.o.}f_1} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_1}, T_{\text{e2e}}^{\text{l.o.}f_1}}$	$R_{\text{e2e}}^{\text{l.o.}f_1}$	$\begin{aligned} &= \bigwedge_{i \in \{2,3\}} \left(R_{s_i} - r_{s_i}^{x(f_1)} \right) \\ &= (20-10) \wedge (20-10) \\ &= 10 \end{aligned}$
	$T_{\text{e2e}}^{\text{l.o.}f_1}$	$\begin{aligned} &= \sum_{i \in \{2,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{\text{e2e}}^{\text{l.o.}f_1}} \right) \\ &= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{850 + 10 \cdot 20}{10} \\ &= 40 + \frac{1300}{10} \\ &= 170 \end{aligned}$
	$=$	$= \beta_{10,170}$
D^{f_1}		$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_1} &= b^{f_1} \\ 10 \cdot [t - 170]^+ &= 25 \\ t &= 172\frac{1}{2} \end{aligned}$
B^{f_1}		$\begin{aligned} \alpha^{f_1}\left(T_{\text{e2e}}^{\text{l.o.}f_1} \right) &= 5 \cdot 170 + 25 \\ &= 875 \end{aligned}$

`PmooArrivalBound.java`

<table>
<tr><th colspan="2">PMOO</th><th>Arbitrary Multiplexing</th></tr>
<tr><td rowspan="2">s_2</td><td>$\alpha_{s_2}^{x(f_1)}$</td><td rowspan="2">$$\begin{aligned} &= \alpha_{s_2}^{f_2} + \alpha_{s_2}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{10,50} \end{aligned}$$</td></tr>
<tr><td>$\alpha_{s_2}^{\bar{x}(f_1)}$</td></tr>
<tr><td rowspan="2">s_3</td><td>$\alpha_{s_3}^{x(f_1)}$</td><td rowspan="2">$$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,412\frac{1}{2}} \\ &= \gamma_{10,870\frac{5}{6}} \end{aligned}$$</td></tr>
<tr><td>$\alpha_{s_3}^{\bar{x}(f_1)}$</td></tr>
<tr><td rowspan="3">$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}}$</td><td>$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}$</td><td>$$\begin{aligned} &= \bigwedge_{i\in\{2,3\}} \left(R_{s_i} - r_{s_i}^{x(f_1)}\right) \\ &= (20-10) \wedge (20-10) \\ &= 10 \end{aligned}$$</td></tr>
<tr><td>$T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}$</td><td>$$\begin{aligned} &= \sum_{i\in\{2,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}}\right) \\ &= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{870\frac{5}{6} + 10 \cdot 20}{10} \\ &= 40 + \frac{1320\frac{5}{6}}{10} \\ &= 172\frac{1}{12} \end{aligned}$$</td></tr>
<tr><td>$=$</td><td>$= \beta_{10,172\frac{1}{12}}$</td></tr>
<tr><td colspan="2">D^{f_1}</td><td>$$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} &= b^{f_1} \\ 10 \cdot \left[t - 172\frac{1}{12}\right]^+ &= 25 \\ t &= 174\frac{7}{12} \end{aligned}$$</td></tr>
<tr><td colspan="2">B^{f_1}</td><td>$$\begin{aligned} \alpha^{f_1}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}\right) &= 5 \cdot 172\frac{1}{12} + 25 \\ &= 885\frac{5}{12} \end{aligned}$$</td></tr>
</table>

Flow f_2

Total Flow Analysis

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing
s_2	α_{s_2}	$= \alpha^{f_2}_{s_2} + \alpha^{f_1}_{s_2} + \alpha^{f_3}_{s_2}$ $= \gamma_{5,25} + \gamma_{5,25} + \gamma_{5,25}$ $= \gamma_{15,75}$	
	$D^{f_2}_{s_2}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t-20]^{+} = 75$ $t = 23\frac{3}{4}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t-20]^{+} = 15 \cdot t + 75$ $t = 95$
	$B^{f_2}_{s_2}$	$\alpha_{s_2}\left(T_{s_2}\right) = 15 \cdot 20 + 75$ $= 375$	
s_1	α_{s_1}	$= \alpha^{f_2}_{s_1} + \alpha^{f_0}_{s_1}$ $= \gamma_{5,137\frac{1}{2}} + \gamma_{5,159\frac{3}{8}}$ $= \gamma_{10,296\frac{7}{8}}$	$= \alpha^{f_2}_{s_1} + \alpha^{f_0}_{s_1}$ $= \gamma_{5,250} + \gamma_{5,241\frac{2}{3}}$ $= \gamma_{10,491\frac{2}{3}}$
	$D^{f_2}_{s_1}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t-20]^{+} = 296\frac{7}{8}$ $t = 34\frac{27}{32}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t-20]^{+} = 10 \cdot t + 491\frac{2}{3}$ $t = 89\frac{1}{6}$
	$B^{f_2}_{s_1}$	$\alpha_{s_1}\left(T_{s_1}\right) = 10 \cdot 20 + 296\frac{7}{8}$ $= 496\frac{7}{8}$	$\alpha_{s_1}\left(T_{s_1}\right) = 10 \cdot 20 + 491\frac{2}{3}$ $= 691\frac{2}{3}$
D^{f_2}		$D^{f_2}_{s_2} + D^{f_2}_{s_1} = 58\frac{19}{32}$	$D^{f_2}_{s_2} + D^{f_2}_{s_1} = 184\frac{1}{6}$
B^{f_2}		$\max\left\{B^{f_2}_{s_2}, B^{f_2}_{s_1}\right\} = 496\frac{7}{8}$	$\max\left\{B^{f_2}_{s_2}, B^{f_2}_{s_1}\right\} = 691\frac{2}{3}$

Separate Flow Analysis and PMOO Analysis

Analyses

SFA			FIFO Multiplexing	Arbitrary Multiplexing
s_2	$\alpha^{x(f_2)}_{s_2}$		$= \alpha^{f_1}_{s_2} + \alpha^{f_3}_{s_2}$ $= \gamma_{5,25} + \gamma_{5,25}$ $= \gamma_{10,50}$	
	$\beta^{\text{l.o.}f_2}_{s_2} = \beta_{s_2} \ominus \alpha^{x(f_2)}_{s_2}$		$= \beta_{10,22\frac{1}{2}}$	$= \beta_{10,45}$
s_1	$\alpha^{x(f_2)}_{s_1}$		$= \alpha^{f_0}_{s_1} = \gamma_{5,159\frac{3}{8}}$	$= \alpha^{f_0}_{s_1} = \gamma_{5,241\frac{2}{3}}$
	$\beta^{\text{l.o.}f_2}_{s_1} = \beta_{s_1} \ominus \alpha^{x(f_2)}_{s_1}$ $= \beta_{R^{\text{l.o.}f_2}_{s_1}, T^{\text{l.o.}f_2}_{s_1}}$	$R^{\text{l.o.}f_2}_{s_1}$	$\left[R_{s_2} - r^{x(f_2)}_{s_2}\right]^{+} = 15$	
		$T^{\text{l.o.}f_2}_{s_1}$	$\beta_{s_1} = b^{x(f_2)}_{s_1}$ $20 \cdot [t-20]^{+} = 159\frac{3}{8}$ $t = 27\frac{31}{32}$	$\beta_{s_1} = \alpha^{x(f_2)}_{s_1}$ $20 \cdot [t-20]^{+} = 5 \cdot t + 241\frac{2}{3}$ $t = 42\frac{7}{9}$
		$=$	$= \beta_{10,27\frac{31}{32}}$	$= \beta_{10,42\frac{7}{9}}$
$\beta^{\text{l.o.}f_2}_{\text{e2e}} = \beta_{R^{\text{l.o.}f_2}_{\text{e2e}}, T^{\text{l.o.}f_2}_{\text{e2e}}}$			$\bigotimes_{i=1}^{2} \beta^{\text{l.o.}f_2}_{s_i} = \beta_{10,50\frac{15}{32}}$	$\bigotimes_{i=1}^{2} \beta^{\text{l.o.}f_2}_{s_i} = \beta_{10,87\frac{7}{9}}$
D^{f_2}			$\beta^{\text{l.o.}f_2}_{\text{e2e}} = b^{f_2}$ $10 \cdot \left[t - 50\frac{15}{32}\right]^{+} = 25$ $t = 52\frac{31}{32}$	$\beta^{\text{l.o.}f_2}_{\text{e2e}} = b^{f_2}$ $10 \cdot \left[t - 87\frac{7}{9}\right]^{+} = 25$ $t = 90\frac{5}{18}$
B^{f_2}			$\alpha^{f_2}\left(T^{\text{l.o.}f_2}_{\text{e2e}}\right) = 5 \cdot 50\frac{15}{32} + 25$ $= 277\frac{11}{32}$	$\alpha^{f_2}\left(T^{\text{l.o.}f_2}_{\text{e2e}}\right) = 5 \cdot 87\frac{7}{9} + 25$ $= 463\frac{8}{9}$

PMOO		Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_2)}$	$\begin{aligned} &= \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{10,50} \end{aligned}$
	$\alpha_{s_2}^{\bar{x}(f_2)}$	
s_1	$\alpha_{s_1}^{x(f_2)}$	$= \alpha_{s_1}^{f_0} = \gamma_{5,241\frac{2}{3}}$
	$\alpha_{s_1}^{\bar{x}(f_2)}$	
$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_2} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_2}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_2}}$	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_2}$	$\begin{aligned} &= \bigwedge_{i \in \{2,1\}} \left(R_{s_i} - r_{s_i}^{x(f_2)} \right) \\ &= (20-10) \wedge (20-5) \\ &= 10 \end{aligned}$
	$T_{\mathrm{e2e}}^{\mathrm{l.o.}f_2}$	$\begin{aligned} &= \sum_{i \in \{2,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_2)} + r_{s_i}^{x(f_2)} \cdot T_{s_i}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_2}} \right) \\ &= 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{241\frac{2}{3} + 5 \cdot 20}{10} \\ &= 40 + \frac{591\frac{2}{3}}{10} \\ &= 99\frac{1}{6} \end{aligned}$
	$=$	$= \beta_{10,99\frac{1}{6}}$
D^{f_2}		$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_2} &= b^{f_2} \\ 10 \cdot \left[t - 99\frac{1}{6} \right] &= 25 \\ t &= 101\frac{2}{3} \end{aligned}$
B^{f_2}		$\begin{aligned} \alpha^{f_2}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_2} \right) &= 5 \cdot 99\frac{1}{6} + 25 \\ &= 520\frac{5}{6} \end{aligned}$

Flow f_3

Total Flow Analysis

Analysis

`PbooArrivalBound_Concatenation.java`

TFA		FIFO Multiplexing	Arbitrary Multiplexing	
		`PbooArrivalBound_Concatenation.java`		`PmooArrivalBound.java`
s_2	α_{s_2}	$\begin{aligned} &= \alpha_{s_2}^{f_3} + \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2} \\ &= \gamma_{5,25} + \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{15,75} \end{aligned}$		
	$D_{s_2}^{f_3}$	$\begin{aligned} \beta_{s_2} &= b_{s_2} \\ 20 \cdot [t-20]^{+} &= 75 \\ t &= 23\frac{3}{4} \end{aligned}$	$\begin{aligned} \beta_{s_2} &= \alpha_{s_2} \\ 20 \cdot [t-20]^{+} &= 15 \cdot t + 75 \\ t &= 95 \end{aligned}$	
	$B_{s_2}^{f_3}$	$\begin{aligned} \alpha_{s_2}\left(T_{s_2}\right) &= 15 \cdot 20 + 75 \\ &= 375 \end{aligned}$		
s_0	α_{s_0}	$\begin{aligned} &= \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,137\frac{1}{2}} \\ &= \gamma_{10,162\frac{1}{2}} \end{aligned}$	$\begin{aligned} &= \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_3} \\ &= \gamma_{5,25} + \gamma_{5,250} \\ &= \gamma_{10,275} \end{aligned}$	
	$D_{s_0}^{f_3}$	$\begin{aligned} \beta_{s_0} &= b_{s_0} \\ 20 \cdot [t-20]^{+} &= 162\frac{1}{2} \\ t &= 28\frac{1}{8} \end{aligned}$	$\begin{aligned} \beta_{s_0} &= \alpha_{s_0} \\ 20 \cdot [t-20]^{+} &= 10 \cdot t + 275 \\ t &= 67\frac{1}{2} \end{aligned}$	
	$B_{s_0}^{f_3}$	$\begin{aligned} \alpha_{s_0}\left(T_{s_0}\right) &= 10 \cdot 20 + 162\frac{1}{2} \\ &= 262\frac{1}{2} \end{aligned}$	$\begin{aligned} \alpha_{s_0}\left(T_{s_0}\right) &= 10 \cdot 20 + 275 \\ &= 475 \end{aligned}$	
s_3	α_{s_3}	$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}} \\ &= \gamma_{15,675} \end{aligned}$	$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,391\frac{2}{3}} \\ &= \gamma_{15,1100} \end{aligned}$	$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} + \alpha_{s_3}^{f_3} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,412\frac{1}{2}} \\ &= \gamma_{15,1120\frac{5}{6}} \end{aligned}$
	$D_{s_3}^{f_3}$	$\begin{aligned} \beta_{s_3} &= b_{s_3} \\ 20 \cdot [t-20]^{+} &= 675 \\ t &= 53\frac{3}{4} \end{aligned}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3} \\ 20 \cdot [t-20]^{+} &= 15 \cdot t + 1100 \\ t &= 300 \end{aligned}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3} \\ 20 \cdot [t-20]^{+} &= 15 \cdot t + 1120\frac{5}{6} \\ t &= 304\frac{1}{6} \end{aligned}$
	$B_{s_3}^{f_3}$	$\begin{aligned} \alpha_{s_3}\left(T_{s_3}\right) &= 15 \cdot 20 + 675 \\ &= 975 \end{aligned}$	$\begin{aligned} \alpha_{s_3}\left(T_{s_3}\right) &= 15 \cdot 20 + 1100 \\ &= 1400 \end{aligned}$	$\begin{aligned} \alpha_{s_3}\left(T_{s_3}\right) &= 15 \cdot 20 + 1120\frac{5}{6} \\ &= 1420\frac{5}{6} \end{aligned}$
D^{f_3}		$D_{s_2}^{f_3} + D_{s_0}^{f_3} + D_{s_3}^{f_3} = 105\frac{5}{8}$	$D_{s_2}^{f_3} + D_{s_0}^{f_3} + D_{s_3}^{f_3} = 462\frac{1}{2}$	$D_{s_2}^{f_3} + D_{s_0}^{f_3} + D_{s_3}^{f_3} = 466\frac{2}{3}$
B^{f_3}		$\max\left\{B_{s_2}^{f_3}, B_{s_0}^{f_3}, B_{s_3}^{f_3}\right\} = 975$	$\max\left\{B_{s_2}^{f_3}, B_{s_0}^{f_3}, B_{s_3}^{f_3}\right\} = 1400$	$\max\left\{B_{s_2}^{f_3}, B_{s_0}^{f_3}, B_{s_3}^{f_3}\right\} = 1420\frac{5}{6}$

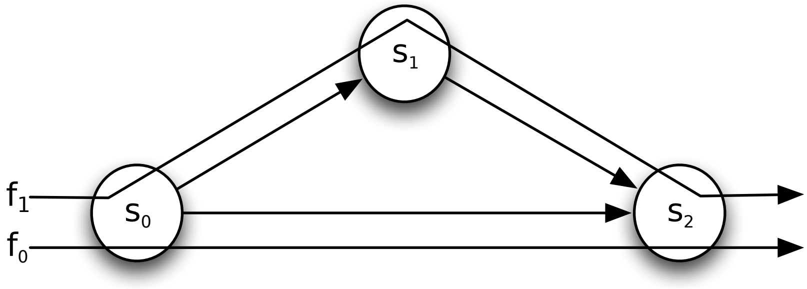
Separate Flow Analysis and PMOO Analysis

Analyses

	SFA		FIFO Multiplexing	Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_3)}$		$\begin{aligned} &= \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2} \\ &= \gamma_{5,25} + \gamma_{5,25} \\ &= \gamma_{10,50} \end{aligned}$	
	$\beta_{s_2}^{\text{l.o.}f_3} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_3)}$		$= \beta_{10,22\frac{1}{2}}$	$= \beta_{10,45}$
s_0	$\alpha_{s_0}^{x(f_3)}$		$= \alpha_{s_0}^{f_0} = \gamma_{5,25}$	
	$\beta_{s_0}^{\text{l.o.}f_3} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_3)}$		$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s_3	$\alpha_{s_3}^{x(f_3)}$		$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} \\ &= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}} \\ &= \gamma_{10,431\frac{1}{4}} \end{aligned}$	$\begin{aligned} &= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} \\ &= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} \\ &= \gamma_{10,708\frac{1}{3}} \end{aligned}$
	$\begin{aligned} \beta_{s_3}^{\text{l.o.}f_3} &= \beta_{s_3} \ominus \alpha_{s_3}^{x(f_3)} \\ &= \beta_{R_{s_3}^{\text{l.o.}f_3}, T_{s_3}^{\text{l.o.}f_3}} \end{aligned}$	$R_{s_3}^{\text{l.o.}f_3}$	$\left[R_{s_3} - r_{s_3}^{x(f_3)}\right]^+ = 10$	
		$T_{s_3}^{\text{l.o.}f_3}$	$\begin{aligned} \beta_{s_3} &= b_{s_3}^{x(f_3)} \\ 20 \cdot [t-20]^+ &= 431\frac{1}{4} \\ t &= 41\frac{9}{16} \end{aligned}$	$\begin{aligned} \beta_{s_3} &= \alpha_{s_3}^{x(f_3)} \\ 20 \cdot [t-20]^+ &= 10 \cdot t + 708\frac{1}{3} \\ t &= 110\frac{5}{6} \end{aligned}$
		$=$	$= \beta_{10,41\frac{9}{16}}$	$= \beta_{10,110\frac{5}{6}}$
$\beta_{\text{e2e}}^{\text{l.o.}f_3} = \beta_{R_{\text{e2e}}^{\text{l.o.}f_3}, T_{\text{e2e}}^{\text{l.o.}f_3}}$			$\bigotimes_{i=\{2,0,3\}} \beta_{s_i}^{\text{l.o.}f_3} = \beta_{10,85\frac{5}{16}}$	$\bigotimes_{i=\{2,0,3\}} \beta_{s_i}^{\text{l.o.}f_3} = \beta_{10,184\frac{1}{6}}$
D^{f_3}			$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_3} &= b^{f_3} \\ 10 \cdot \left[t - 85\frac{5}{16}\right]^+ &= 25 \\ t &= 87\frac{13}{16} \end{aligned}$	$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o.}f_3} &= b^{f_3} \\ 10 \cdot \left[t - 184\frac{1}{6}\right]^+ &= 25 \\ t &= 186\frac{2}{3} \end{aligned}$
B^{f_3}			$\begin{aligned} \alpha^{f_3}\left(T_{\text{e2e}}^{\text{l.o.}f_3}\right) &= 5 \cdot 85\frac{5}{16} + 25 \\ &= 451\frac{9}{16} \end{aligned}$	$\begin{aligned} \alpha^{f_3}\left(T_{\text{e2e}}^{\text{l.o.}f_3}\right) &= 5 \cdot 184\frac{1}{6} + 25 \\ &= 945\frac{5}{6} \end{aligned}$

PMOO		Arbitrary Multiplexing
s_2	$\alpha_{s_2}^{x(f_3)}$	$= \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2} = \gamma_{5,25} + \gamma_{5,25} = \gamma_{10,50}$
	$\alpha_{s_2}^{\bar{x}(f_3)}$	
s_0	$\alpha_{s_0}^{x(f_3)}$	$= \alpha_{s_0}^{f_0} = \gamma_{5,25}$
	$\alpha_{s_0}^{\bar{x}(f_3)}$	
s_3	$\alpha_{s_3}^{x(f_3)}$	$= \alpha_{s_3}^{f_0} + \alpha_{s_3}^{f_1} = \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} = \gamma_{10,708\frac{1}{3}}$
	$\alpha_{s_3}^{\bar{x}(f_3)}$	
$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_3} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_3}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_3}}$	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_3}$	$= \bigwedge_{i \in \{2,0,3\}} \left(R_{s_i} - r_{s_i}^{x(f_3)} \right) = (20-10) \wedge (20-5) \wedge (20-10) = 10$
	$T_{\mathrm{e2e}}^{\mathrm{l.o.}f_3}$	$= \sum_{i \in \{2,0,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_3)} + r_{s_i}^{x(f_3)} \cdot T_{s_i}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_3}} \right) = 20 + \frac{50 + 10 \cdot 20}{10} + 20 + \frac{25 + 5 \cdot 20}{10} + 20 + \frac{708\frac{1}{3} + 10 \cdot 20}{10} = 60 + \frac{1283\frac{1}{3}}{10} = 188\frac{1}{3}$
	$=$	$= \beta_{10,188\frac{1}{3}}$
D^{f_3}		$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_3} = b^{f_3}$; $10 \cdot \left[t - 188\frac{1}{3} \right]^+ = 25$; $t = 190\frac{5}{6}$
B^{f_3}		$\alpha^{f_3}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_3} \right) = 5 \cdot 188\frac{1}{3} + 25 = 966\frac{2}{3}$

FeedForward_1SC_2Flows_1AC_2Paths



$\mathbb{S} = \{s_0, s_1, s_2\}$ with

$$\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20},\ i \in \{0,1,2\}$$

$\mathbb{F} = \{f_0, f_1\}$ with

$$\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25},\ n \in \{0,1\}$$

Flow f_0

Total Flow Analysis

Arrival Bounds

$(s_1, \{f_1\}, \emptyset) =: \alpha_{s_1}^{f_1}$ $(s_2, \{f_0\}, \emptyset) =: \alpha_{s_2}^{f_0}$ $=: \alpha_{s_i}^{f_n}$ with $(n,i) \in \{(1,1),(0,2)\}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_0}^{\mathrm{l.o.}f_n}$			
$\alpha_{s_0}^{x(f_n)}$		$= \gamma_{5,25}$	
$\beta_{s_0}^{\mathrm{l.o.}f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_n)}$ $= \beta_{R_{s_0}^{\mathrm{l.o.}f_n}, T_{s_0}^{\mathrm{l.o.}f_n}}$	$R_{s_0}^{\mathrm{l.o.}f_n}$	$= 15$	
	$T_{s_0}^{\mathrm{l.o.}f_n}$	$\beta_{s_0} = b_{s_0}^{x(f_n)}$ $20 \cdot [t-20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_n)}$ $20 \cdot [t-20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
$\alpha_{s_i}^{f_n} = \alpha^{f_n} \oslash \beta_{s_0}^{\mathrm{l.o.}f_n}$ $= \gamma_{r_{s_i}^{f_n}, b_{s_i}^{f_n}}$	$r_{s_i}^{f_n}$	$= 5$	
	$b_{s_i}^{f_n}$	$\alpha^{f_0}\left(T_{s_0}^{\mathrm{l.o.}f_n}\right) = 5 \cdot 21\frac{1}{4} + 25$ $= 131\frac{1}{4}$	$\alpha^{f_0}\left(T_{s_0}^{\mathrm{l.o.}f_n}\right) = 5 \cdot 28\frac{1}{3} + 25$ $= 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$

$(s_2, \{f_1\}, \emptyset) =: \alpha_{s_2}^{f_1}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha_{s_2}^{f_1} = \alpha^{f_1} \oslash \left(\beta_{s_0}^{\mathrm{l.o.}f_1} \otimes \beta_{s_1}^{\mathrm{l.o.}f_1}\right)$ (reuse of previous result) $= \alpha^{f_1} \oslash \beta_{s_0}^{\mathrm{l.o.}f_1} \oslash \beta_{s_1}^{\mathrm{l.o.}f_1}$ $= \alpha_{s_1}^{f_1} \oslash \beta_{s_1}^{\mathrm{l.o.}f_1}$			
$\alpha_{s_1}^{x(f_1)}$		$= \gamma_{0,0}$	
$\beta_{s_1}^{\mathrm{l.o.}f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)}$		$= \beta_{s_1} = \beta_{20,20}$	
$\alpha_{s_1}^{f_1}$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\alpha_{s_2}^{f_1} = \alpha^{f_1} \oslash \beta_{s_1}^{\mathrm{l.o.}f_1}$ $= \gamma_{r_{s_2}^{f_1}, b_{s_2}^{f_1}}$	$r_{s_2}^{f_1}$	$= 5$	
	$b_{s_2}^{f_1}$	$\alpha^{f_0}\left(T_{s_0}^{\mathrm{l.o.}f_0}\right) = 5 \cdot 20 + 131\frac{1}{4}$ $= 231\frac{1}{4}$	$\alpha^{f_0}\left(T_{s_0}^{\mathrm{l.o.}f_0}\right) = 5 \cdot 20 + 166\frac{2}{3}$ $= 266\frac{2}{3}$
	$=$	$= \gamma_{5,231\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}}$

Remark:
`PmooArrivalBound.java` will have the same result as `PbooArrivalBound_Concatenation.java` because f_0 does not have cross-traffic interfering on multiple consecutive hops.

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing
s_0	α_{s_0}	$= \alpha^{f_0} + \alpha^{f_1}$ $= \gamma_{5,25} + \gamma_{5,25}$ $= \gamma_{10,50}$	
	$D^{f_0}_{s_0}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t-20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}$ $20 \cdot [t-20]^+ = 10 \cdot t + 50$ $t = 45$
	$B^{f_0}_{s_0}$	$\alpha_{s_0}(T_{s_0}) = 10 \cdot 20 + 50$ $= 250$	
s_2	α_{s_2}	$= \alpha^{f_0}_{s_2} + \alpha^{f_1}_{s_2}$ $= \gamma_{5,231\frac{1}{4}} + \gamma_{5,131\frac{1}{4}}$ $= \gamma_{10,362\frac{1}{2}}$	$= \alpha^{f_0}_{s_2} + \alpha^{f_1}_{s_2}$ $= \gamma_{5,266\frac{2}{3}} + \gamma_{5,166\frac{2}{3}}$ $= \gamma_{10,433\frac{1}{3}}$
	$D^{f_0}_{s_2}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t-20]^+ = 362\frac{1}{2}$ $t = 38\frac{1}{8}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t-20]^+ = 10 \cdot t + 433\frac{1}{3}$ $t = 83\frac{1}{3}$
	$B^{f_0}_{s_2}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 362\frac{1}{2}$ $= 562\frac{1}{2}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 433\frac{1}{3}$ $= 633\frac{1}{3}$
D^{f_0}		$\sum_{i=\{0,2\}} D^{f_0}_{s_i} = 60\frac{5}{8}$	$\sum_{i=\{0,2\}} D^{f_0}_{s_i} = 128\frac{1}{3}$
B^{f_0}		$\max_{i=\{0,2\}} B^{f_0}_{s_i} = 562\frac{1}{2}$	$\max_{i=\{0,2\}} B^{f_0}_{s_i} = 633\frac{1}{3}$

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$(s_2, \{f_1\}, f_0) =: \alpha^{f_1}_{s_2}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_1}_{s_2} = \alpha^{f_1} \oslash \left(\beta^{\text{l.o.}f_1}_{s_0} \otimes \beta^{\text{l.o.}f_1}_{s_1}\right)$			
$\alpha^{x(f_0)}_{s_0}$		$= \gamma_{5,25}$	
$\beta^{\text{l.o.}f_1}_{s_0} = \beta_{s_0} \ominus \alpha^{x(f_1)}_{s_0}$ $= \beta_{R^{\text{l.o.}f_1}_{s_0}, T^{\text{l.o.}f_1}_{s_0}}$	$R^{\text{l.o.}f_1}_{s_0}$	$= 15$	
	$T^{\text{l.o.}f_1}_{s_0}$	$\beta_{s_0} = b^{x(f_1)}_{s_0}$ $20 \cdot [t-20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_0} = \alpha^{x(f_1)}_{s_0}$ $20 \cdot [t-20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
$\alpha^{x(f_1)}_{s_1}$		$= \gamma_{0,0}$	
$\beta^{\text{l.o.}f_1}_{s_1} = \beta_{s_1} \ominus \alpha^{x(f_1)}_{s_1}$		$= \beta_{s_1} = \beta_{20,20}$	
$\beta^{\text{l.o.}f_1}_{s_0} \otimes \beta^{\text{l.o.}f_1}_{s_1} = \beta^{\text{l.o.}f_1}_{\langle s_0,s_1\rangle}$ $= \beta_{R^{\text{l.o.}f_1}_{\langle s_0,s_1\rangle}, T^{\text{l.o.}f_1}_{\langle s_0,s_1\rangle}}$		$= \beta^{\text{l.o.}f_1}_{s_0} \otimes \beta_{s_1}$ $= \beta_{15,21\frac{1}{4}} \otimes \beta_{20,20}$ $= \beta_{15,41\frac{1}{4}}$	$= \beta^{\text{l.o.}f_1}_{s_0} \otimes \beta_{s_1}$ $= \beta_{15,28\frac{1}{3}} \otimes \beta_{20,20}$ $= \beta_{15,48\frac{1}{3}}$
$\alpha^{f_1}_{s_2} = \alpha^{f_1} \oslash \left(\beta^{\text{l.o.}f_1}_{s_0} \otimes \beta^{\text{l.o.}f_1}_{s_1}\right)$ $= \gamma_{r^{f_1}_{\langle s_0,s_1\rangle}, b^{f_1}_{\langle s_0,s_1\rangle}}$	$r^{f_1}_{\langle s_0,s_1\rangle}$	$= 5$	
	$b^{f_1}_{\langle s_0,s_1\rangle}$	$\alpha^{f_1}\left(T^{\text{l.o.}f_1}_{\langle s_0,s_1\rangle}\right) = 5 \cdot 41\frac{1}{4} + 25$ $= 231\frac{1}{4}$	$\alpha^{f_1}\left(T^{\text{l.o.}f_1}_{\langle s_0,s_1\rangle}\right) = 5 \cdot 48\frac{1}{3} + 25$ $= 266\frac{2}{3}$
	$=$	$= \gamma_{5,231\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}}$

Remark:
`PmooArrivalBound.java` will have the same result as `PbooArrivalBound_Concatenation.java` because f_0 does not have cross-traffic interfering on multiple consecutive hops.

Analyses

SFA			FIFO Multiplexing	Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_0)}$		$= \alpha^{f_1} = \gamma_{5,25}$	
	$\begin{aligned}\beta_{s_0}^{\mathrm{l.o.}f_0} &= \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} \\ &= \beta_{R_{s_0}^{\mathrm{l.o.}f_0}, T_{s_0}^{\mathrm{l.o.}f_0}}\end{aligned}$	$R_{s_0}^{\mathrm{l.o.}f_0}$	$\left[R_{s_0} - r_{s_0}^{x(f_0)}\right]^+ = 15$	
		$T_{s_0}^{\mathrm{l.o.}f_0}$	$\begin{aligned}\beta_{s_0} &= b_{s_0}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 25 \\ t &= 21\frac{1}{4}\end{aligned}$	$\begin{aligned}\beta_{s_0} &= \alpha_{s_0}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 25 \\ t &= 28\frac{1}{3}\end{aligned}$
		$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s_2	$\alpha_{s_2}^{x(f_0)}$		$= \gamma_{5,231\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}}$
	$\begin{aligned}\beta_{s_2}^{\mathrm{l.o.}f_0} &= \beta_{s_2} \ominus \alpha_{s_2}^{x(f_0)} \\ &= \beta_{R_{s_2}^{\mathrm{l.o.}f_0}, T_{s_2}^{\mathrm{l.o.}f_0}}\end{aligned}$	$R_{s_2}^{\mathrm{l.o.}f_0}$	$\left[R_{s_2} - r_{s_2}^{x(f_0)}\right]^+ = 15$	
		$T_{s_2}^{\mathrm{l.o.}f_0}$	$\begin{aligned}\beta_{s_2} &= b_{s_2}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 231\frac{1}{4} \\ t &= 31\frac{9}{16}\end{aligned}$	$\begin{aligned}\beta_{s_2} &= \alpha_{s_2}^{x(f_0)} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 266\frac{2}{3} \\ t &= 44\frac{4}{9}\end{aligned}$
		$=$	$= \beta_{15,31\frac{9}{16}}$	$= \beta_{15,44\frac{4}{9}}$
$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}$			$\bigotimes_{i=\{0,2\}} \beta_{s_i}^{\mathrm{l.o.}f_0} = \beta_{15,52\frac{13}{16}}$	$\bigotimes_{i=\{0,2\}} \beta_{s_i}^{\mathrm{l.o.}f_0} = \beta_{15,72\frac{7}{9}}$
D^{f_0}			$\begin{aligned}\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} &= b^{f_0} \\ 15 \cdot \left[t - 52\frac{13}{16}\right]^+ &= 25 \\ t &= 54\frac{23}{48}\end{aligned}$	$\begin{aligned}\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} &= b^{f_0} \\ 15 \cdot \left[t - 72\frac{7}{9}\right]^+ &= 25 \\ t &= 74\frac{4}{9}\end{aligned}$
B^{f_0}			$\begin{aligned}\alpha^{f_0}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}\right) &= 5 \cdot 52\frac{13}{16} + 25 \\ &= 289\frac{1}{16}\end{aligned}$	$\begin{aligned}\alpha^{f_0}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}\right) &= 5 \cdot 72\frac{7}{9} + 25 \\ &= 388\frac{8}{9}\end{aligned}$

PMOO		Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{5,25}$
	$\alpha_{s_0}^{\bar{x}(f_0)}$	
s_2	$\alpha_{s_2}^{x(f_0)}$	$= \gamma_{5,266\frac{2}{3}}$
	$\alpha_{s_2}^{\bar{x}(f_0)}$	
$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}$	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}$	$\begin{aligned} &= \bigwedge_{i\in\{0,2\}} \left(R_{s_i} - r_{s_i}^{x(f_0)}\right) \\ &= (20-5) \wedge (20-5) \\ &= 15\end{aligned}$
	$T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}$	$\begin{aligned} &= \sum_{i\in\{0,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}}\right) \\ &= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{266\frac{2}{3} + 5 \cdot 20}{15} \\ &= 40 + \frac{491\frac{2}{3}}{15} \\ &= 72\frac{7}{9}\end{aligned}$
	$=$	$= \beta_{15,72\frac{7}{9}}$
D^{f_0}		$\begin{aligned}\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_0} &= b^{f_0} \\ 15 \cdot \left[t - 72\frac{7}{9}\right]^+ &= 25 \\ t &= 74\frac{4}{9}\end{aligned}$
B^{f_0}		$\begin{aligned}\alpha^{f_0}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_0}\right) &= 5 \cdot 72\frac{7}{9} + 25 \\ &= 388\frac{8}{9}\end{aligned}$

Flow f_1

Total Flow Analysis

Arrival Bounds

$(s_1, \{f_1\}, \emptyset) =: \alpha^{f_1}_{s_1}$ $(s_2, \{f_0\}, \emptyset) =: \alpha^{f_0}_{s_2}$ $=: \alpha^{f_n}_{s_i}$ with $(n,i) \in \{(1,1),(0,2)\}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_n}_{s_i} = \alpha^{f_n} \oslash \beta^{\text{l.o.}f_n}_{s_0}$			
$\alpha^{x(f_n)}_{s_0}$		$= \gamma_{5,25}$	
$\beta^{\text{l.o.}f_n}_{s_0} = \beta_{s_0} \ominus \alpha^{x(f_n)}_{s_0}$ $= \beta_{R^{\text{l.o.}f_n}_{s_0}, T^{\text{l.o.}f_n}_{s_0}}$	$R^{\text{l.o.}f_n}_{s_0}$	$= 15$	
	$T^{\text{l.o.}f_n}_{s_0}$	$\beta_{s_0} = b^{x(f_n)}_{s_0}$ $20 \cdot [t-20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_0} = \alpha^{x(f_n)}_{s_0}$ $20 \cdot [t-20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
$\alpha^{f_n}_{s_i} = \alpha^{f_n} \oslash \beta^{\text{l.o.}f_n}_{s_0}$	$r^{f_n}_{s_i}$	$= 5$	
	$b^{f_n}_{s_i}$	$\alpha^{f_0}\left(T^{\text{l.o.}f_n}_{s_0}\right) = 131\frac{1}{4}$	$\alpha^{f_0}\left(T^{\text{l.o.}f_n}_{s_0}\right) = 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$

$(s_2, \{f_1\}, \emptyset) =: \alpha^{f_1}_{s_2}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_1}_{s_2} = \alpha^{f_1} \oslash \left(\beta^{\text{l.o.}f_1}_{s_0} \otimes \beta^{\text{l.o.}f_1}_{s_1}\right) = \alpha^{f_1} \oslash \beta^{\text{l.o.}f_1}_{s_0} \oslash \beta^{\text{l.o.}f_1}_{s_1} = \alpha^{f_1}_{s_1} \oslash \beta^{\text{l.o.}f_1}_{s_1}$			
$\alpha^{x(f_1)}_{s_1}$		$= \gamma_{0,0}$	
$\beta^{\text{l.o.}f_1}_{s_1} = \beta_{s_1} \ominus \alpha^{x(f_1)}_{s_1}$		$= \beta_{s_1} = \beta_{20,20}$	
$\alpha^{f_1}_{s_1}$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\alpha^{f_1}_{s_2} = \alpha^{f_1}_{s_1} \oslash \beta^{\text{l.o.}f_1}_{s_1}$ $= \alpha^{f_1} \oslash \beta^{\text{l.o.}f_1}_{s_1}$	$r^{f_1}_{s_2}$	$= 5$	
	$b^{f_1}_{s_2}$	$\alpha^{f_0}\left(T^{\text{l.o.}f_0}_{s_0}\right) = 231\frac{1}{4}$	$\alpha^{f_0}\left(T^{\text{l.o.}f_0}_{s_0}\right) = 266\frac{2}{3}$
	$=$	$= \gamma_{5,231\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}}$

Remark:
`PmooArrivalBound.java` will have the same result as `PbooArrivalBound_Concatenation.java` because f_1 does not have cross-traffic interfering on multiple consecutive hops.

Analysis

TFA		FIFO Multiplexing	Arbitrary Multiplexing
s_0	α_{s_0}	$= \alpha^{f_0} + \alpha^{f_1} = \gamma_{5,25} + \gamma_{5,25} = \gamma_{10,50}$	
	$D^{f_0}_{s_0}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t-20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}$ $20 \cdot [t-20]^+ = 10 \cdot t + 50$ $t = 45$
	$B^{f_0}_{s_0}$	$\alpha_{s_0}(T_{s_0}) = 10 \cdot 20 + 50$ $= 250$	
s_1	α_{s_1}	$= \alpha^{f_1}_{s_1} = \gamma_{5,131\frac{1}{4}}$	$= \alpha^{f_1}_{s_1} = \gamma_{5,166\frac{2}{3}}$
	$D^{f_1}_{s_1}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t-20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	FIFO per micro flow $\beta_{s_1} = b_{s_1}$ $20 \cdot [t-20]^+ = 166\frac{2}{3}$ $t = 28\frac{1}{3}$
	$B^{f_1}_{s_1}$	$\alpha_{s_1}(T_{s_1}) = 5 \cdot 20 + 131\frac{1}{4}$ $= 231\frac{1}{4}$	$\alpha_{s_1}(T_{s_1}) = 5 \cdot 20 + 166\frac{2}{3}$ $= 266\frac{2}{3}$
s_2	α_{s_2}	$= \alpha^{f_0}_{s_2} + \alpha^{f_1}_{s_2}$ $= \gamma_{5,231\frac{1}{4}} + \gamma_{5,131\frac{1}{4}}$ $= \gamma_{10,362\frac{1}{2}}$	$= \alpha^{f_0}_{s_2} + \alpha^{f_1}_{s_2}$ $= \gamma_{5,266\frac{2}{3}} + \gamma_{5,166\frac{2}{3}}$ $= \gamma_{10,433\frac{1}{3}}$
	$D^{f_1}_{s_2}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t-20]^+ = 362\frac{1}{2}$ $t = 38\frac{1}{8}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t-20]^+ = 10 \cdot t + 433\frac{1}{3}$ $t = 83\frac{1}{3}$
	$B^{f_1}_{s_2}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 362\frac{1}{2}$ $= 562\frac{1}{2}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 433\frac{1}{3}$ $= 633\frac{1}{3}$
D^{f_1}		$\sum_{i=0}^{2} \beta^{f_1}_{s_i} = 87\frac{3}{16}$	$\sum_{i=0}^{2} \beta^{f_1}_{s_i} = 156\frac{2}{3}$
B^{f_1}		$\max_{i=0}^{2} B^{f_1}_{s_i} = 562\frac{1}{2}$	$\max_{i=0}^{2} B^{f_1}_{s_i} = 633\frac{1}{3}$

Separate Flow Analysis and PMOO Analysis

Arrival Bounds

$(s_2, \{f_0\}, \emptyset) =: \alpha^{f_0}_{s_2}$		FIFO Multiplexing	Arbitrary Multiplexing
$\alpha^{f_0}_{s_2} = \alpha^{f_0} \oslash \beta^{\text{l.o.}f_0}_{s_0}$			
$\alpha^{x(f_0)}_{s_0}$		$= \gamma_{5,25}$	
$\beta^{\text{l.o.}f_0}_{s_0} = \beta_{s_0} \ominus \alpha^{x(f_0)}_{s_0}$ $= \beta_{R^{\text{l.o.}f_0}_{s_0}, T^{\text{l.o.}f_0}_{s_0}}$	$R^{\text{l.o.}f_0}_{s_0}$	$= 15$	
	$T^{\text{l.o.}f_0}_{s_0}$	$\beta_{s_0} = b^{x(f_0)}_{s_0}$ $20 \cdot [t-20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_0} = \alpha^{x(f_0)}_{s_0}$ $20 \cdot [t-20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$=\beta_{15,21\frac{1}{4}}$	$=\beta_{15,28\frac{1}{3}}$
$\alpha^{f_0}_{s_2} = \alpha^{f_0} \oslash \beta^{\text{l.o.}f_0}_{s_2}$ $= \alpha^{f_0} \oslash \beta^{\text{l.o.}f_0}_{s_2}$	$r^{f_0}_{s_2}$	$= 5$	
	$b^{f_0}_{s_2}$	$\alpha^{f_0}\left(T^{\text{l.o.}f_0}_{s_0}\right) = 131\frac{1}{4}$	$\alpha^{f_0}\left(T^{\text{l.o.}f_0}_{s_0}\right) = 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$

Remark:
`PmooArrivalBound.java` will have the same result as `PbooArrivalBound_Concatenation.java` because f_1 does not have cross-traffic interfering on multiple consecutive hops.

Analyses

<table>
<tr><td colspan="3">SFA</td><td>FIFO Multiplexing</td><td>Arbitrary Multiplexing</td></tr>
<tr><td rowspan="4">s_0</td><td colspan="2">$\alpha_{s_0}^{x(f_1)}$</td><td colspan="2">$= \alpha^{f_0} = \gamma_{5,25}$</td></tr>
<tr><td rowspan="3">$\beta_{s_0}^{\mathrm{l.o.}f_1} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_1)}$</td><td>$R_{s_0}^{\mathrm{l.o.}f_1}$</td><td colspan="2">$\left[R_{s_0} - r_{s_0}^{x(f_1)}\right]^+ = 15$</td></tr>
<tr><td>$T_{s_0}^{\mathrm{l.o.}f_1}$</td><td>$\begin{aligned} \beta_{s_0} &= b_{s_0}^{x(f_1)} \\ 20 \cdot [t-20]^+ &= 25 \\ t &= 21\frac{1}{4} \end{aligned}$</td><td>$\begin{aligned} \beta_{s_0} &= \alpha_{s_0}^{x(f_1)} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 25 \\ t &= 28\frac{1}{3} \end{aligned}$</td></tr>
<tr><td>$=$</td><td>$= \beta_{15,21\frac{1}{4}}$</td><td>$= \beta_{15,28\frac{1}{3}}$</td></tr>
<tr><td rowspan="2">s_1</td><td colspan="2">$\alpha_{s_1}^{x(f_1)}$</td><td colspan="2">$= \gamma_{0,0}$</td></tr>
<tr><td colspan="2">$\beta_{s_1}^{\mathrm{l.o.}f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)}$</td><td colspan="2">$= \beta_{s_1} = \beta_{20,20}$</td></tr>
<tr><td rowspan="4">s_2</td><td colspan="2">$\alpha_{s_2}^{x(f_1)}$</td><td>$= \gamma_{5,131\frac{1}{4}}$</td><td>$= \gamma_{5,166\frac{2}{3}}$</td></tr>
<tr><td rowspan="3">$\beta_{s_2}^{\mathrm{l.o.}f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)}$</td><td>$R_{s_2}^{\mathrm{l.o.}f_1}$</td><td colspan="2">$\left[R_{s_2} - r_{s_2}^{x(f_1)}\right]^+ = 15$</td></tr>
<tr><td>$T_{s_2}^{\mathrm{l.o.}f_1}$</td><td>$\begin{aligned} \beta_{s_2} &= b_{s_2}^{x(f_1)} \\ 20 \cdot [t-20]^+ &= 131\frac{1}{4} \\ t &= 26\frac{9}{16} \end{aligned}$</td><td>$\begin{aligned} \beta_{s_2} &= \alpha_{s_2}^{x(f_1)} \\ 20 \cdot [t-20]^+ &= 5 \cdot t + 166\frac{2}{3} \\ t &= 37\frac{7}{9} \end{aligned}$</td></tr>
<tr><td>$=$</td><td>$= \beta_{15,26\frac{9}{16}}$</td><td>$= \beta_{15,37\frac{7}{9}}$</td></tr>
<tr><td colspan="3">$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}}$</td><td>$\bigotimes_{i=0}^{2} \beta_{s_i}^{\mathrm{l.o.}f_1} = \beta_{15,67\frac{13}{16}}$</td><td>$\bigotimes_{i=0}^{2} \beta_{s_i}^{\mathrm{l.o.}f_1} = \beta_{15,86\frac{1}{9}}$</td></tr>
<tr><td colspan="3">D^{f_1}</td><td>$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} &= b^{f_1} \\ 15 \cdot \left[t - 67\frac{13}{16}\right]^+ &= 25 \\ t &= 69\frac{23}{48} \end{aligned}$</td><td>$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} &= b^{f_1} \\ 15 \cdot \left[t - 86\frac{1}{9}\right]^+ &= 25 \\ t &= 87\frac{7}{9} \end{aligned}$</td></tr>
<tr><td colspan="3">B^{f_1}</td><td>$\begin{aligned} \alpha^{f_1}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}\right) &= 5 \cdot 67\frac{13}{16} + 25 \\ &= 364\frac{1}{16} \end{aligned}$</td><td>$\begin{aligned} \alpha^{f_1}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}\right) &= 5 \cdot 86\frac{1}{9} + 25 \\ &= 455\frac{5}{9} \end{aligned}$</td></tr>
</table>

PMOO		Arbitrary Multiplexing
s_0	$\alpha_{s_0}^{x(f_1)}$ $\alpha_{s_0}^{\bar{x}(f_1)}$	$= \gamma_{5,25}$
s_1	$\alpha_{s_0}^{x(f_1)}$ $\alpha_{s_0}^{\bar{x}(f_1)}$	$= \gamma_{0,0}$
s_2	$\alpha_{s_2}^{x(f_1)}$ $\alpha_{s_2}^{\bar{x}(f_1)}$	$= \gamma_{5,166\frac{2}{3}}$
$\beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} = \beta_{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}, T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}}$	$R_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}$	$\begin{aligned} &= \bigwedge_{i \in \{0,1,2\}} \left(R_{s_i} - r_{s_i}^{x(f_1)} \right) \\ &= (20-5) \wedge (20-0) \wedge (20-5) \\ &= 15 \end{aligned}$
	$T_{\mathrm{e2e}}^{\mathrm{l.o.}f_3}$	$\begin{aligned} &= \sum_{i \in \{0,1,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{\mathrm{e2e}}^{\mathrm{l.o.}f_1}} \right) \\ &= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{0 + 0 \cdot 20}{15} + 20 + \frac{166\frac{2}{3} + 5 \cdot 20}{15} \\ &= 60 + \frac{391\frac{2}{3}}{15} \\ &= 86\frac{1}{9} \end{aligned}$
	$=$	$= \beta_{15,86\frac{1}{9}}$
D^{f_1}		$\begin{aligned} \beta_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} &= b^{f_1} \\ 15 \cdot \left[t - 86\frac{1}{9} \right]^+ &= 25 \\ t &= 87\frac{7}{9} \end{aligned}$
B^{f_1}		$\begin{aligned} \alpha^{f_1}\left(T_{\mathrm{e2e}}^{\mathrm{l.o.}f_1} \right) &= 5 \cdot 86\frac{1}{9} + 25 \\ &= 455\frac{5}{9} \end{aligned}$