

Network Calculus Tests – Tandem Network Configurations

Version 1.1 (2014-Dec-30)



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General Information

- The network calculus analyses presented in this document were created for the purpose of testing the Disco Deterministic Network Calculator (DiscoDNC)¹ – an open-source deterministic network calculus tool developed by the *Distributed Computer Systems (DISCO) Lab* at the University of Kaiserslautern.
- Naming of the individual network configurations depicts the name of the according functional test for the DiscoDNC.
- The naming scheme used in this document is detailed in NetworkCalculus_NamingScheme.pdf.
- Arrival bound computations are equivalent to the `PbooArrivalBound_Output_PerHop.java` class of the DiscoDNC.
- The end-to-end left-over service curve for PBOO arrival bounds can be computed by simply convolving the server-local ones.
- Arrival bounds for `PmooArrivalBound.java` and analyses using them are listed only if results are different to PBOO.

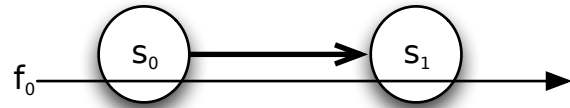
Changelog:

Version 1.1 (2014-Dec-30):

- Streamlined the PMOO left-over latency $T_{e2e}^{l.o.f}$ computation.
- Adapted to naming scheme version 1.1.

¹<http://disco.cs.uni-kl.de/index.php/projects/disco-dnc>

Tandem_1SC_1Flow



- $\beta_{s_0} = \beta_{s_1} = \beta_{R_{s_i}, T_{s_i}} = \beta_{10,10}, i \in \{0,1\}$
- $\mathcal{F} = \{f_0\}$
- $\alpha^{f_0} = \gamma_{r^{f_0}, b^{f_0}} = \gamma_{5,25}$

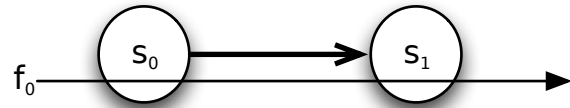
$\text{arrivalBound}(s_1, \{f_0\}, \mathcal{G}), \mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_0}$		$= \gamma_{5,25}$	
$\alpha_{s_0}^{x(f_0)}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$		$= \beta_{10,10}$	
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0} = \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$	$r_{s_1}^{f_0}$	$= 5$	
	$b_{s_1}^{f_0}$	$\alpha^{f_0}(T_{s_0}^{\text{l.o.}f_0}) = 5 \cdot 10 + 25 = 75$	
	$=$	$= \gamma_{5,75}$	

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha_{s_0}^{f_0}$	$= \gamma_{5,25}$	
	$D_{s_0}^{f_0}$	$\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 25$ $t = 12\frac{1}{2}$	FIFO per micro flow $\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 25$ $t = 12\frac{1}{2}$
	$B_{s_0}^{f_0}$	$\alpha_{s_0}(T_{s_0}) = 5 \cdot 10 + 25$ $= 75$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0}$	$= \gamma_{5,75}$	
	$D_{s_1}^{f_0}$	$\beta_{s_1} = b_{s_1}$ $10 \cdot [t - 10]^+ = 75$ $t = 17\frac{1}{2}$	FIFO per micro flow $\beta_{s_1} = b_{s_1}$ $10 \cdot [t - 10]^+ = 75$ $t = 17\frac{1}{2}$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 5 \cdot 10 + 75$ $= 125$	
D^{f_0}		$\sum_{i=0}^1 D_{s_i}^{f_0} = 30$	
B^{f_0}		$\max_{i=\{0,1\}} b_{s_i}^{f_0} = 125$	

SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{s_0}^{1.o.f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{s_0}$	$= \beta_{10,10}$	
s_1	$\alpha_{s_1}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{s_1}^{1.o.f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)} = \beta_{s_1}$	$= \beta_{10,10}$	
$\beta_{e2e}^{1.o.f_0} = \beta_{R_{e2e}^{1.o.f_0}, T_{e2e}^{1.o.f_0}}$		$\bigotimes_{i=0}^1 \beta_{s_i}^{1.o.f_0} = \beta_{10,20}$	
D^{f_0}		$\beta_{e2e}^{1.o.f_0} = b^{f_0}$ $10 \cdot [t - 20]^+ = 25$ $t = 22\frac{1}{2}$	
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{1.o.f_0}) = 5 \cdot 20 + 25$ $= 125$	

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$
s_1	$\alpha_{s_0}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$
$\beta_{e2e}^{1.o.f_0} = \beta_{R_{e2e}^{1.o.f_0}, T_{e2e}^{1.o.f_0}}$	$R_{e2e}^{1.o.f_0} = \bigwedge_{i \in \{0,1\}} (R_{s_i} - r_{s_i}^{x(f_0)})$	$= (10 - 0) \wedge (10 - 0)$ $= 10$
	$T_{e2e}^{1.o.f_0} = \sum_{i \in \{0,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{1.o.f_0}} \right)$	$= 10 + \frac{0 + 0 \cdot 10}{10} + 10 + \frac{0 + 0 \cdot 10}{10}$ $= 20$
	$=$	$= \beta_{10,20}$
D^{f_0}		$\beta_{e2e}^{1.o.f_0} = b^{f_0}$ $10 \cdot [t - 20]^+ = 25$ $t = 22\frac{1}{2}$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{1.o.f_0}) = 5 \cdot 20 + 25$ $= 125$

Tandem_2SCs_1Flow



- $\beta_{s_0} = \beta_{R_{s_0}, T_{s_0}} = \beta_{10,10}$
- $\beta_{s_1} = \beta_{R_{s_1}, T_{s_1}} = \beta_{6,6}$
- $\mathcal{F} = \{f_0\}$
- $\alpha^{f_0} = \gamma_{r^{f_0}, b^{f_0}} = \gamma_{5,25}$

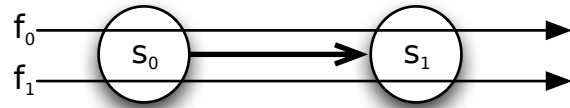
$\text{arrivalBound}(s_1, \{f_0\}, \mathcal{G}), \mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_0}$		$= \gamma_{5,25}$	
$\alpha_{s_0}^{x(f_0)}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{\text{l.o.}f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{R_{s_0}^{\text{l.o.}f_0}, T_{s_0}^{\text{l.o.}f_0}}$		$= \beta_{10,10}$	
$\alpha_{s_1}^{f_0} = \alpha^{f_0} \oslash \beta_{s_0}^{\text{l.o.}f_0} = \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$	$r_{s_1}^{f_0}$	$= 5$	
	$b_{s_1}^{f_0}$	$\alpha^{f_0}(T_{s_0}^{\text{l.o.}f_0}) = 5 \cdot 10 + 25 = 75$	
	$=$	$= \gamma_{5,75}$	

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha_{s_0}^{f_0}$	$= \gamma_{5,25}$	
	$D_{s_0}^{f_0}$	$\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 25$ $t = 12\frac{1}{2}$	FIFO per micro flow $\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 25$ $t = 12\frac{1}{2}$
	$B_{s_0}^{f_0}$	$\alpha_{s_0}(T_{s_0}) = 5 \cdot 10 + 25$ $= 75$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0}$	$= \gamma_{5,75}$	
	$D_{s_1}^{f_0}$	$\beta_{s_1} = b_{s_1}$ $6 \cdot [t - 6]^+ = 75$ $t = 18\frac{1}{2}$	FIFO per micro flow $\beta_{s_1} = b_{s_1}$ $6 \cdot [t - 6]^+ = 75$ $t = 18\frac{1}{2}$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 5 \cdot 6 + 75$ $= 105$	
D^{f_0}		$\sum_{i=0}^1 D_{s_i}^{f_0} = 31$	
B^{f_0}		$\max_{i=\{0,1\}} b_{s_i}^{f_0} = 105$	

SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{s_0}^{1.o.f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{s_0}$	$= \beta_{10,10}$	
$s_0 s_1$	$\alpha_{s_0 s_1}^{x(f_0)}$	$= \gamma_{0,0}$	
s_1	$\alpha_{s_1}^{x(f_0)} = \alpha_{s_0 s_1}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{s_1}^{1.o.f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)} = \beta_{s_1}$	$= \beta_{6,6}$	
$\beta_{e2e}^{1.o.f_0} = \beta_{R_{e2e}^{1.o.f_0}, T_{e2e}^{1.o.f_0}}$		$\bigotimes_{i=0}^1 \beta_{s_i}^{1.o.f_0} = \beta_{6,16}$	
D^{f_0}		$\beta_{e2e}^{1.o.f_0} = b^{f_0}$ $6 \cdot [t - 16]^+ = 25$ $t = 20\frac{1}{6}$	
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{1.o.f_0}) = 5 \cdot 16 + 25$ $= 105$	

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$
s_1	$\alpha_{s_1}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$	$R_{e2e}^{l.o.f_0} = \bigwedge_{i \in \{0,1\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$	$= (10 - 0) \wedge (6 - 0)$
		$= 6$
	$T_{e2e}^{l.o.f_0} = \sum_{i \in \{0,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 10 + \frac{0 + 0 \cdot 10}{6} + 6 + \frac{0 + 0 \cdot 6}{6}$
	$=$	$= \frac{16}{6}$
D^{f_0}		$\beta_{e2e}^{l.o.f_0} = b^{f_0}$
		$6 \cdot [t - 16]^+ = 25$
		$t = 20 \frac{1}{6}$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 16 + 25$
		$= 105$

Tandem_1SC_2Flows_1AC_1Path



- $\beta_{s_0} = \beta_{s_1} = \beta_{R_{s_i}, T_{s_i}} = \beta_{10,10}, i \in \{0,1\}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0,1\}$

$\text{arrivalBound}(s_1, \{f_0\}, \{f_1\}) = \alpha_{s_1}^{f_0}$ $= \text{arrivalBound}(s_1, \{f_1\}, \{f_0\}) = \alpha_{s_1}^{f_1}$	FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_n}$		$= \gamma_{5,25}$
$\alpha_{s_0}^{x f_n}$		$= \gamma_{0,0}$
$\beta_{s_0}^{\text{l.o.} f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x f_n} = \beta_{R_{s_0}^{\text{l.o.} f_n}, T_{s_0}^{\text{l.o.} f_n}}$		$= \beta_{10,10}$
$\alpha_{s_1}^{f_n} = \alpha_{s_0}^{f_n} \odot \beta_{s_0}^{\text{l.o.} f_n} = \gamma_{r_{s_1}^{f_n}, b_{s_1}^{f_n}}$	$r_{s_1}^{f_n}$	$= 5$
	$b_{s_1}^{f_n}$	$\alpha_{s_0}^{f_0}(T_{s_0}^{\text{l.o.} f_0}) = 5 \cdot 10 + 25 = 75$
	$=$	$= \gamma_{5,75}$

$\text{arrivalBound}(s_1, \{f_0, f_1\}, \{\}) = \alpha_{s_1}^{\{f_0, f_1\}}$	FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{\{f_0, f_1\}}$		$= \gamma_{10,50}$
$\alpha_{s_0}^{x \{f_0, f_1\}}$		$= \gamma_{0,0}$
$\beta_{s_0}^{\text{l.o.} \{f_0, f_1\}} = \beta_{s_0} \ominus \alpha_{s_0}^{x \{f_0, f_1\}} = \beta_{R_{s_0}^{\text{l.o.} \{f_0, f_1\}}, T_{s_0}^{\text{l.o.} \{f_0, f_1\}}}$		$= \beta_{10,10}$
$\alpha_{s_1}^{\{f_0, f_1\}} = \alpha_{s_0}^{\{f_0, f_1\}} \odot \beta_{s_0}^{\text{l.o.} \{f_0, f_1\}} = \gamma_{r_{s_1}^{\{f_0, f_1\}}, b_{s_1}^{\{f_0, f_1\}}}$	$r_{s_1}^{\{f_0, f_1\}}$	$= 10$
	$b_{s_1}^{\{f_0, f_1\}}$	$\alpha_{s_0}^{\{f_0, f_1\}}(T_{s_0}^{\text{l.o.} \{f_0, f_1\}}) = 10 \cdot 10 + 50 = 150$
	$=$	$= \gamma_{10,150}$

Flows $f_n, n \in \{0, 1\}$

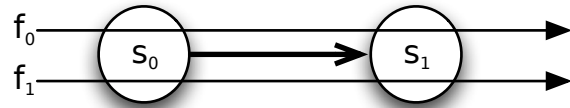
TFA results will be equal for all flows as they share the same path of servers.

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha^{f_0} + \alpha^{f_1}$	$= \gamma_{10,50}$	
	$D_{s_0}^{f_n}$	$\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 50$ $t = 15$	$\beta_{s_0} = \alpha_{s_0}$ $10 \cdot [t - 10]^+ = 10 \cdot t + 50$ $0 \cdot t = 150$ $\Rightarrow D_{s_0}^{f_n} = \infty$
	$B_{s_0}^{f_n}$	$\alpha_{s_0}(T_{s_0}) = 10 \cdot 10 + 50$ $= 150$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{\{f_0, f_1\}}$	$= \gamma_{10,150}$	
	$D_{s_1}^{f_n}$	$\beta_{s_1} = b_{s_1}$ $10 \cdot [t - 10]^+ = 150$ $t = 25$	$\beta_{s_1} = \alpha_{s_1}$ $10 \cdot [t - 10]^+ = 10 \cdot t + 150$ $0 \cdot t = 250$ $\Rightarrow D_{s_1}^{f_n} = \infty$
	$B_{s_1}^{f_n}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 10 + 150$ $= 250$	
D^{f_n}		$\sum_{i=0}^1 D_{s_i}^{f_n} = 40$	$\sum_{i=0}^1 D_{s_i}^{f_n} = \infty$
B^{f_n}		$\max_{i=\{0,1\}} b_{s_i}^{f_n} = 250$	

SFA			FIFO_MUX	ARB_MUX
s0	$\alpha_{s_0}^{xf_n}$		$= \gamma_{5,25}$	
	$\beta_{s_0}^{l.o.f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_n)}$	$R_{s_0}^{l.o.f_n}$	$= 5$	
		$T_{s_0}^{l.o.f_n}$	$\beta_{s_0} = b_{s_0}^{xf_n}$ $10 \cdot [t - 10]^+ = 25$ $t = 12\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}^{xf_n}$ $10 \cdot [t - 10]^+ = 5 \cdot t + 25$ $t = 25$
		$=$	$= \beta_{5,12\frac{1}{2}}$	$= \beta_{5,25}$
s1	$\alpha_{s_1}^{xf_n}$		$= \gamma_{5,75}$	
	$\beta_{s_1}^{l.o.f_n} = \beta_{s_1} \ominus \alpha_{s_1}^{xf_n}$	$R_{s_1}^{l.o.f_n}$	$= 5$	
		$T_{s_1}^{l.o.f_n}$	$\beta_{s_1} = b_{s_1}^{xf_n}$ $10 \cdot [t - 10]^+ = 75$ $t = 17\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}^{xf_n}$ $10 \cdot [t - 10]^+ = 5 \cdot t + 75$ $t = 35$
		$=$	$= \beta_{5,17\frac{1}{2}}$	$= \beta_{5,35}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$		$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_n} = \beta_{5,30}$	$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_n} = \beta_{5,60}$	
D^{f_n}		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $5 \cdot [t - 30]^+ = 25$ $t = 35$	$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $5 \cdot [t - 60]^+ = 25$ $t = 65$	
B^{f_n}		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 30 + 25$ $= 175$	$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 60 + 25$ $= 325$	

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}f_n}$	$= \gamma_{5,25}$
	$\alpha_{s_0}^{xf_n}$	$= \gamma_{5,25}$
s_1	$\alpha_{s_1}^{\bar{x}f_n}$	$= \gamma_{0,0}$
	$\alpha_{s_1}^{xf_n}$	$= \gamma_{5,75}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$	$R_{e2e}^{l.o.f_n} = \bigwedge_{i \in \{0,1\}} (R_{s_i} - r_{s_i}^{xf_n})$	$= (10 - 5) \wedge (10 - 5)$
		$= 5$
	$T_{e2e}^{l.o.f_n} = \sum_{i \in \{0,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}f_n} + r_{s_i}^{xf_n} \cdot T_{s_i}}{R_{e2e}^{l.o.f_n}} \right)$	$= 10 + \frac{25 + 5 \cdot 10}{5} + 10 + \frac{0 + 5 \cdot 10}{5}$
		$= 45$
	$=$	$= \beta_{5,45}$
D^{f_n}		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $5 \cdot [t - 45]^+ = 25$ $t = 50$
B^{f_n}		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 45 + 25$ $= 250$

Tandem_2SCs_2Flows_1AC_1Path



- $\beta_{s_0} = \beta_{R_{s_0}, T_{s_0}} = \beta_{10,10}$
- $\beta_{s_1} = \beta_{R_{s_1}, T_{s_1}} = \beta_{6,6}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{2\frac{1}{2}, 12\frac{1}{2}}, n \in \{0, 1\}$

$\text{arrivalBound}(s_1, \{f_0\}, \{f_1\}) = \alpha_{s_1}^{f_0}$ $= \text{arrivalBound}(s_1, \{f_1\}, \{f_0\}) = \alpha_{s_1}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_n}$		$= \gamma_{2\frac{1}{2}, 12\frac{1}{2}}$	
$\alpha_{s_0}^{x f_n}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{\text{l.o.} f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x f_n} = \beta_{R_{s_0}^{\text{l.o.} f_n}, T_{s_0}^{\text{l.o.} f_n}}$		$= \beta_{10,10}$	
$\alpha_{s_1}^{f_n} = \alpha_{s_0}^{f_n} \oslash \beta_{s_0}^{\text{l.o.} f_n} = \gamma_{r_{s_1}^{f_n}, b_{s_1}^{f_n}}$	$r_{s_1}^{f_n}$	$= 2\frac{1}{2}$	
	$b_{s_1}^{f_n}$	$\alpha_{s_0}^{f_n}(T_{s_0}^{\text{l.o.} f_n}) = 2\frac{1}{2} \cdot 10 + 12\frac{1}{2} = 37\frac{1}{2}$	
	$=$	$= \gamma_{2\frac{1}{2}, 37\frac{1}{2}}$	

$\text{arrivalBound}(s_1, \{f_0, f_1\}, \{\}) = \alpha_{s_1}^{\{f_0, f_1\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{\{f_0, f_1\}}$		$= \gamma_{5,25}$	
$\alpha_{s_0}^{x \{f_0, f_1\}}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{\text{l.o.} \{f_0, f_1\}} = \beta_{s_0} \ominus \alpha_{s_0}^{x \{f_0, f_1\}} = \beta_{R_{s_0}^{\text{l.o.} \{f_0, f_1\}}, T_{s_0}^{\text{l.o.} \{f_0, f_1\}}}$		$= \beta_{10,10}$	
$\alpha_{s_1}^{\{f_0, f_1\}} = \alpha_{s_0}^{\{f_0, f_1\}} \oslash \beta_{s_0}^{\text{l.o.} \{f_0, f_1\}} = \gamma_{r_{s_1}^{\{f_0, f_1\}}, b_{s_1}^{\{f_0, f_1\}}}$	$r_{s_1}^{\{f_0, f_1\}}$	$= 5$	
	$b_{s_1}^{\{f_0, f_1\}}$	$\alpha_{s_0}^{\{f_0, f_1\}}(T_{s_0}^{\text{l.o.} \{f_0, f_1\}}) = 5 \cdot 10 + 25 = 75$	
	$=$	$= \gamma_{5,75}$	

Flows f_n , $n \in \{0, 1\}$

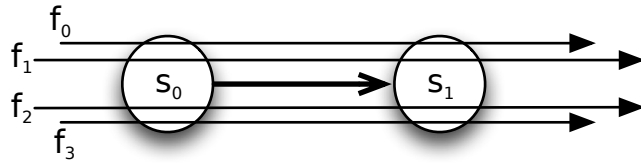
TFA results will be equal for all flows as they share the same path of servers.

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha^{f_0} + \alpha^{f_1}$	$= \gamma_{5,25}$	
	$D_{s_0}^{f_n}$	$\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 25$ $t = 12\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}$ $10 \cdot [t - 10]^+ = 5 \cdot t + 25$ $t = 25$
	$B_{s_0}^{f_n}$	$\alpha_{s_0}(T_{s_0}) = 5 \cdot 10 + 25$ $= 75$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{\{f_0, f_1\}}$	$= \gamma_{5,75}$	
	$D_{s_1}^{f_n}$	$\beta_{s_1} = b_{s_1}$ $6 \cdot [t - 6]^+ = 75$ $t = 18\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $6 \cdot [t - 6]^+ = 5 \cdot t + 75$ $t = 111$
	$B_{s_1}^{f_n}$	$\alpha_{s_1}(T_{s_1}) = 5 \cdot 6 + 75$ $= 105$	
D^{f_n}		$\sum_{i=0}^1 D_{s_i}^{f_n} = 31$	$\sum_{i=0}^1 D_{s_i}^{f_n} = 136$
B^{f_n}		$\max_{i=\{0,1\}} b_{s_i}^{f_n} = 105$	

SFA			FIFO_MUX	ARB_MUX
s0	$\alpha_{s_0}^{xf_n}$		$= \gamma_{2\frac{1}{2}, 12\frac{1}{2}}$	
	$\beta_{s_0}^{l.o.f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{xf_n}$	$R_{s_0}^{l.o.f_n}$	$= 7\frac{1}{2}$	
		$T_{s_0}^{l.o.f_n}$	$\beta_{s_0} = b_{s_0}^{xf_n}$ $10 \cdot [t - 10]^+ = 12\frac{1}{2}$ $t = 11\frac{1}{4}$	$\beta_{s_0} = \alpha_{s_0}^{xf_n}$ $10 \cdot [t - 10]^+ = 2\frac{1}{2} \cdot t + 12\frac{1}{2}$ $t = 15$
		$=$	$= \beta_{7\frac{1}{2}, 11\frac{1}{4}}$	$= \beta_{7\frac{1}{2}, 15}$
s1	$\alpha_{s_1}^{xf_n}$		$= \gamma_{2\frac{1}{2}, 37\frac{1}{2}}$	
	$\beta_{s_1}^{l.o.f_n} = \beta_{s_1} \ominus \alpha_{s_1}^{xf_n}$	$R_{s_1}^{l.o.f_n}$	$= 3\frac{1}{2}$	
		$T_{s_1}^{l.o.f_n}$	$\beta_{s_1} = b_{s_1}^{xf_n}$ $6 \cdot [t - 6]^+ = 37\frac{1}{2}$ $t = 12\frac{1}{4}$	$\beta_{s_1} = \alpha_{s_1}^{xf_n}$ $6 \cdot [t - 6]^+ = 2\frac{1}{2} \cdot t + 37\frac{1}{2}$ $t = 21$
		$=$	$= \beta_{3\frac{1}{2}, 12\frac{1}{4}}$	$= \beta_{3\frac{1}{2}, 21}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$		$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_n} = \beta_{3\frac{1}{2}, 23\frac{1}{2}}$	$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_n} = \beta_{3\frac{1}{2}, 36}$	
D^{f_n}		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $3\frac{1}{2} \cdot [t - 23\frac{1}{2}]^+ = 12\frac{1}{2}$ $t = 27\frac{1}{14}$	$\beta_{e2e}^{l.o.f_n} = b^{j_i}$ $3\frac{1}{2} \cdot [t - 36]^+ = 12\frac{1}{2}$ $t = 39\frac{4}{7}$	
B^{f_n}		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 2\frac{1}{2} \cdot 23\frac{1}{2} + 12\frac{1}{2}$ $= 71\frac{1}{4}$	$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 2\frac{1}{2} \cdot 36 + 12\frac{1}{2}$ $= 102\frac{1}{2}$	

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}f_n}$	$= \gamma_{2\frac{1}{2}, 12\frac{1}{2}}$
	$\alpha_{s_0}^{xf_n}$	$= \gamma_{2\frac{1}{2}, 12\frac{1}{2}}$
s_1	$\alpha_{s_1}^{\bar{x}f_n}$	$= \gamma_{0,0}$
	$\alpha_{s_1}^{xf_n}$	$= \gamma_{2\frac{1}{2}, 37\frac{1}{2}}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$	$R_{e2e}^{l.o.f_n} = \bigwedge_{i \in \{0,1\}} (R_{s_i} - r_{s_i}^{xf_n})$	$= (10 - 2\frac{1}{2}) \wedge (6 - 2\frac{1}{2})$
	$T_{e2e}^{l.o.f_n} = \sum_{i \in \{0,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}f_n} + r_{s_i}^{xf_n} \cdot T_{s_i}}{R_{e2e}^{l.o.f_n}} \right)$	$= 3\frac{1}{2}$
	$=$	$= 6 + \frac{12\frac{1}{2} + 2\frac{1}{2} \cdot 10}{3\frac{1}{2}} + 10 + \frac{0 + 2\frac{1}{2} \cdot 10}{3\frac{1}{2}}$
		$= 31$
	$=$	$= \beta_{3\frac{1}{2}, 31}$
Df_n		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $3\frac{1}{2} \cdot [t - 31]^+ = 12\frac{1}{2}$ $t = 34\frac{4}{7}$
Bf_n		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 2\frac{1}{2} \cdot 31 + 12\frac{1}{2}$ $= 90$

Tandem_1SCs_4Flows_1ACs_1Path



- $\beta_{s_0} = \beta_{s_1} = \beta_{R_{s_i}, T_{s_i}} = \beta_{10,10}, i \in \{0,1\}$
- $\mathcal{F} = \{f_0, f_1, f_2, f_3\}$
- $\alpha^{f_n} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{2,10}, n \in \{0,1,2,3\}$

$\text{arrivalBound}(s_1, xf_n, \{f_n\}) = \alpha_{s_1}^{xf_n}, n \in \{0, 1, 2, 3\}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{xf_n}$		$= \gamma_{6,30}$	
$\alpha_{s_0}^{xxf_n}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{l.o.xf_n} = \beta_{s_0} \ominus \alpha_{s_0}^{xxf_n} = \beta_{R_{s_0}^{l.o.xf_n}, T_{s_0}^{l.o.xf_n}}$		$= \beta_{10,10}$	
$\alpha_{s_1}^{xf_n} = \alpha_{s_0}^{xf_n} \oslash \beta_{s_0}^{l.o.xf_n} = \gamma_{r_{s_1}^{xf_n}, b_{s_1}^{xf_n}}$	$r_{s_1}^{xf_n}$	$= 6$	
	$b_{s_1}^{xf_n}$	$\alpha_{s_0}^{xf_n}(T_{s_0}^{l.o.xf_n}) = 6 \cdot 10 + 30 = 90$	
	$=$	$= \gamma_{6,90}$	

$\text{arrivalBound}(s_1, \{f_0, f_1, f_2, f_3\}, \{\}) = \alpha_{s_1}^{\{f_0, f_1, f_2, f_3\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{\{f_0, f_1, f_2, f_3\}}$		$= \gamma_{8,40}$	
$\alpha_{s_0}^{x\{f_0, f_1, f_2, f_3\}}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{l.o.\{f_0, f_1, f_2, f_3\}} = \beta_{s_0} \ominus \alpha_{s_0}^{x\{f_0, f_1, f_2, f_3\}} = \beta_{R_{s_0}^{l.o.\{f_0, f_1, f_2, f_3\}}, T_{s_0}^{l.o.\{f_0, f_1, f_2, f_3\}}}$		$= \beta_{10,10}$	
$\alpha_{s_1}^{\{f_0, f_1, f_2, f_3\}} = \alpha_{s_0}^{\{f_0, f_1, f_2, f_3\}} \oslash \beta_{s_0}^{l.o.\{f_0, f_1, f_2, f_3\}} = \gamma_{r_{s_1}^{\{f_0, f_1, f_2, f_3\}}, b_{s_1}^{\{f_0, f_1, f_2, f_3\}}}$	$r_{s_1}^{\{f_0, f_1, f_2, f_3\}}$	$= 8$	
	$b_{s_1}^{\{f_0, f_1, f_2, f_3\}}$	$\alpha_{s_0}^{\{f_0, f_1, f_2, f_3\}}(T_{s_0}^{l.o.\{f_0, f_1, f_2, f_3\}}) = 8 \cdot 10 + 40 = 120$	
	$=$	$= \gamma_{8,120}$	

Flows f_n , $n \in \{0, 1, 2, 3\}$

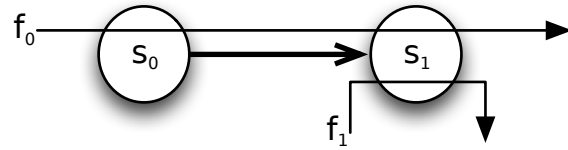
TFA results will be equal for all flows as they share the same path of servers.

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \sum_{n=0}^3 \alpha^{f_n}$	$= \gamma_{8,40}$	
	$D_{s_0}^{f_n}$	$\beta_{s_0} = b_{s_0}$ $10 \cdot [t - 10]^+ = 40$ $t = 14$	$\beta_{s_0} = \alpha_{s_0}$ $10 \cdot [t - 10]^+ = 8 \cdot t + 40$ $t = 70$
	$B_{s_0}^{f_n}$	$\alpha_{s_0}(T_{s_0}) = 8 \cdot 10 + 40$ $= 120$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{\{f_0, f_1, f_2, f_3\}}$	$= \gamma_{8,120}$	
	$D_{s_1}^{f_n}$	$\beta_{s_1} = b_{s_1}$ $10 \cdot [t - 10]^+ = 120$ $t = 22$	$\beta_{s_1} = \alpha_{s_1}$ $10 \cdot [t - 10]^+ = 8 \cdot t + 120$ $t = 110$
	$B_{s_1}^{f_n}$	$\alpha_{s_1}(T_{s_1}) = 8 \cdot 10 + 120$ $= 200$	
D^{f_n}		$\sum_{i=0}^1 D_{s_i}^{f_n} = 36$	$\sum_{i=0}^1 D_{s_i}^{f_n} = 180$
B^{f_n}		$\max_{i=\{0,1\}} b_{s_i}^{f_n} = 200$	

SFA			FIFO_MUX	ARB_MUX
s ₀	$\alpha_{s_0}^{xf_n} = \sum_{k=0}^2 \alpha^{f_k}$		$= \gamma_{6,30}$	
	$\beta_{s_0}^{l.o.f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{xf_n}$	$R_{s_0}^{l.o.f_n}$	$= 4$	
		$T_{s_0}^{l.o.f_n}$	$\beta_{s_0} = b_{s_0}^{xf_n}$ $10 \cdot [t - 10]^+ = 30$ $t = 13$	$\beta_{s_0} = \alpha_{s_0}^{xf_n}$ $10 \cdot [t - 10]^+ = 6 \cdot t + 30$ $t = 32\frac{1}{2}$
		$=$	$= \beta_{4,13}$	$= \beta_{4,32\frac{1}{2}}$
s ₁	$\alpha_{s_1}^{xf_n} = \alpha_{s_1}^{xf_n}$		$= \gamma_{6,90}$	
	$\beta_{s_1}^{l.o.f_n} = \beta_{s_1} \ominus \alpha_{s_1}^{xf_n}$	$R_{s_1}^{l.o.f_n}$	$= 4$	
		$T_{s_1}^{l.o.f_n}$	$\beta_{s_1} = b_{s_1}^{xf_n}$ $10 \cdot [t - 10]^+ = 90$ $t = 19$	$\beta_{s_1} = \alpha_{s_1}^{xf_n}$ $10 \cdot [t - 10]^+ = 4 \cdot t + 90$ $t = 47\frac{1}{2}$
		$=$	$= \beta_{4,19}$	$= \beta_{4,47\frac{1}{2}}$
$\beta_{e2e}^{l.o.f_n}$		$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_n} = \beta_{4,32}$	$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_n} = \beta_{4,80}$	
D^{f_n}		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $4 \cdot [t - 32]^+ = 10$ $t = 34\frac{1}{2}$	$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $4 \cdot [t - 80]^+ = 10$ $t = 82\frac{1}{2}$	
B^{f_n}		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 2 \cdot 32 + 10$ $= 74$	$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 2 \cdot 80 + 10$ $= 170$	

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}f_n}$	$= \gamma_{6,30}$
	$\alpha_{s_0}^{xf_n}$	$= \gamma_{6,30}$
s_1	$\alpha_{s_1}^{\bar{x}f_n}$	$= \gamma_{0,0}$
	$\alpha_{s_1}^{xf_n}$	$= \gamma_{6,90}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$	$R_{e2e}^{l.o.f_n} = \bigwedge_{i \in \{0,1\}} (R_{s_i} - r_{s_i}^{xf_n})$	$= (10 - 6) \wedge (10 - 6)$
	$T_{e2e}^{l.o.f_n} = \sum_{i \in \{0,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}f_n} + r_{s_i}^{xf_n} \cdot T_{s_i}}{R_{e2e}^{l.o.f_n}} \right)$	$= 10 + \frac{30 + 6 \cdot 10}{4} + 10 + \frac{0 + 6 \cdot 10}{4}$
	$=$	$= 37\frac{1}{2}$
D^{f_n}		$= \beta_{3\frac{1}{2},31}$
D^{f_n}		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$
		$4 \cdot [t - 57\frac{1}{2}]^+ = 10$
		$t = 60$
B^{f_n}		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 2 \cdot 57\frac{1}{2} + 10$
		$= 125$

Tandem_1SC_2Flows_1AC_2Paths



- $\beta_{s_0} = \beta_{s_1} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0,1\}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0,1\}$

arrivalBound($s_1, \{f_0\}, \mathcal{G}$) $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_0}$		$= \gamma_{5,25}$	
$\alpha_{s_0}^{x(f_0)}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{l.o.f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{R_{s_0}^{l.o.f_0}, T_{s_0}^{l.o.f_0}}$		$= \beta_{10,10}$	
$\alpha_{s_1}^{f_0} = \alpha_{s_0}^{f_0} \oslash \beta_{s_0}^{l.o.f_0} = \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$	$r_{s_1}^{f_0}$	$= 5$	
	$b_{s_1}^{f_0}$	$\alpha^{f_0}(T_{s_0}^{l.o.f_0}) = 5 \cdot 10 + 25 = 125$	
	$=$	$= \gamma_{5,125}$	

Flow f_0

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha^{f_0}$	$= \gamma_{5,25}$	
	$D_{s_0}^{f_0}$	$\beta_{s_0} = b^{f_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{s_0} = b^{f_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_0}^{f_0}$	$\alpha_{s_0}(T_{s_0}) = 5 \cdot 20 + 25$ $= 125$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0}$	$= \gamma_{5,25} + \gamma_{5,125} = \gamma_{10,150}$	
	$D_{s_1}^{f_0}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 150$ $t = 27\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 150$ $t = 55$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 150$ $= 350$	
D^{f_0}		$\sum_{i=0}^1 D_{s_i}^{f_0} = 48\frac{3}{4}$	$\sum_{i=0}^1 D_{s_i}^{f_0} = 76\frac{1}{4}$
B^{f_0}		$\max_{i=\{0,1\}} b_{s_i}^{f_0} = 350$	

SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{s_0}^{1.o.f_0} = \beta_{s_0}$	$= \beta_{20,20}$	
s_1	$\alpha_{s_1}^{x(f_0)} = \alpha_{s_1}^{f_1}$		$= \gamma_{5,25}$
	$\beta_{s_1}^{1.o.f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{1.o.f_0}$	$= 15$
		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$
		$20 \cdot [t - 20]^+ = 25$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$
		$t = 21\frac{1}{4}$	$t = 28\frac{1}{3}$
	$=$		$= \beta_{15,28\frac{1}{3}}$
$\beta_{e2e}^{1.o.f_0} = \beta_{R_{e2e}^{1.o.f_0}, T_{e2e}^{1.o.f_0}}$		$\bigotimes_{i=0}^1 \beta_{s_i}^{1.o.f_0} = \beta_{15,41\frac{1}{4}}$	$\bigotimes_{i=0}^1 \beta_{s_i}^{1.o.f_0} = \beta_{15,48\frac{1}{3}}$
D^{f_0}		$\beta_{e2e}^{1.o.f_0} = b^{f_0}$	$\beta_{e2e}^{1.o.f_0} = b^{f_0}$
		$15 \cdot [t - 41\frac{1}{4}]^+ = 25$	$15 \cdot [t - 48\frac{1}{3}]^+ = 25$
		$t = 42\frac{11}{12}$	$t = 50$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{1.o.f_0}) = 5 \cdot 41\frac{1}{4} + 25$	$\alpha^{f_0}(T_{e2e}^{1.o.f_0}) = 5 \cdot 48\frac{1}{3} + 25$
		$= 231\frac{1}{4}$	$= 266\frac{2}{3}$

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$
s_1	$\alpha_{s_1}^{\bar{x}(f_0)}$	$= \gamma_{5,25}$
	$\alpha_{s_1}^{x(f_0)}$	$= \gamma_{5,25}$
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$	$R_{e2e}^{l.o.f_0} = \bigwedge_{i \in \{0,1\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$	$= (20 - 5) \wedge (20 - 5)$ $= 15$
	$T_{e2e}^{l.o.f_0} = \sum_{i \in \{0,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 20 + \frac{0 + 0 \cdot 20}{15} + 20 + \frac{25 + 5 \cdot 20}{15}$ $= 48\frac{1}{3}$
	$=$	$= \beta_{15, 48\frac{1}{3}}$
D^{f_0}		$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 48\frac{1}{3}]^+ = 25$ $t = 50$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 48\frac{1}{3} + 25$ $= 266\frac{2}{3}$

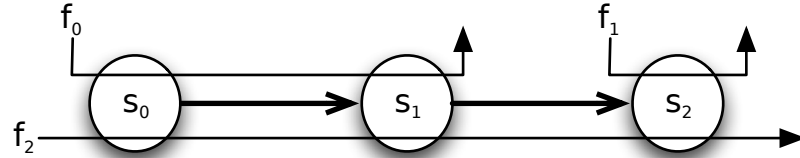
Flow f_1

TFA		FIFO_MUX	ARB_MUX
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_1} + \alpha_{s_1}^{f_0}$	$= \gamma_{5,25} + \gamma_{5,125} = \gamma_{10,150}$	
	$D_{s_1}^{f_1}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 150$ $t = 27\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 150$ $t = 55$
	$B_{s_1}^{f_1}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 150$ $= 350$	
D^{f_1}		$\sum_{i=0}^1 D_{s_i}^{f_1} = 27\frac{1}{2}$	$\sum_{i=0}^1 D_{s_i}^{f_1} = 55$
B^{f_1}		$\max_{i=\{0,1\}} b_{s_i}^{f_1} = 350$	

SFA			FIFO_MUX	ARB_MUX
s ₁	$\alpha_{s_1}^{x(f_1)} = \alpha_{s_1}^{f_0}$		$= \gamma_{5,125}$	
	$\beta_{s_1}^{l.o.f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)} = \beta_{R_{s_1}^{l.o.f_1}, T_{s_1}^{l.o.f_1}}$	$R_{s_1}^{l.o.f_1}$	$= 15$	
		$T_{s_1}^{l.o.f_1}$	$\beta_{s_1} = b_{s_1}^{x(f_1)}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_1)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
		$=$	$= \beta_{15, 26\frac{1}{4}}$	$= \beta_{15, 35}$
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$			$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_1} = \beta_{15, 26\frac{1}{4}}$	$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_1} = \beta_{15, 35}$
D^{f_1}			$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot [t - 26\frac{1}{4}]^+ = 25$ $t = 27\frac{11}{12}$	$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot [t - 35]^+ = 25$ $t = 36\frac{2}{3}$
B^{f_1}			$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 26\frac{1}{4} + 25$ $= 156\frac{1}{4}$	$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 35 + 25$ $= 200$

PMOO		ARB_MUX
s_1	$\alpha_{s_1}^{\bar{x}(f_1)}$	$= \gamma_{5,125}$
	$\alpha_{s_1}^{x(f_1)}$	$= \gamma_{5,125}$
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$	$R_{e2e}^{l.o.f_1} = R_{s_1} - r_{s_1}^{x(f_1)}$	$= 20 - 5$
		$= 15$
	$T_{e2e}^{l.o.f_1} = T_{s_1} + \frac{b_{s_1}^{\bar{x}(f_1)} + r_{s_1}^{x(f_1)} \cdot T_{s_1}}{R_{e2e}^{l.o.f_0}}$	$= 20 + \frac{125 + 5 \cdot 20}{15}$
		$= 35$
	$=$	$= \beta_{15,35}$
D^{f_1}		$\beta_{e2e}^{l.o.f_1} = b^{f_1}$
		$15 \cdot [t - 35]^+ = 25$
		$t = 36\frac{2}{3}$
B^{f_1}		$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 35 + 25$
		$= 200$

Tandem_1SC_3Flows_1AC_3Paths



- $\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0, 1, 2\}$
- $\mathcal{F} = \{f_0, f_1, f_2\}$
- $\alpha^{f_0} = \alpha^{f_1} = \alpha^{f_2} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0, 1, 2\}$

$\text{arrivalBound}(s_1, \{f_0\}, \{f_2\}) = \alpha_{s_1}^{f_0}$ $= \text{arrivalBound}(s_1, \{f_2\}, \{f_0\}) = \alpha_{s_1}^{f_2}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_n}, n \in \{0, 2\}$		$= \gamma_{5,25}$	
$\alpha_{s_0}^{x f_n}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{l.o.f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x f_n} = \beta_{R_{s_0}^{l.o.f_n}, T_{s_0}^{l.o.f_n}}$		$= \beta_{20,20}$	
$\alpha_{s_1}^{f_n} = \alpha_{s_0}^{f_n} \odot \beta_{s_0}^{l.o.f_n} = \gamma_{r_{s_1}^{f_n}, b_{s_1}^{f_n}}$	$r_{s_1}^{f_n}$	$= 5$	
	$b_{s_1}^{f_n}$	$\alpha_{s_0}^{f_n}(T_{s_0}^{l.o.f_n}) = 5 \cdot 20 + 25 = 125$	
	$=$	$= \gamma_{5,125}$	

$\text{arrivalBound}(s_1, \{f_0\}, \{f_0\}) = \alpha_{s_1}^{f_0}$ $= \text{arrivalBound}(s_1, \{f_2\}, \{f_2\}) = \alpha_{s_1}^{f_2}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_n}, n \in \{0, 2\}$		$= \gamma_{5,25}$	
$\alpha_{s_0}^{x f_n}$		$= \gamma_{5,25}$	
$\beta_{s_0}^{l.o.f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x f_n} = \beta_{R_{s_0}^{l.o.f_n}, T_{s_0}^{l.o.f_n}}$	$R_{s_0}^{l.o.f_n}$	$= 15$	
	$T_{s_0}^{l.o.f_n}$	$\beta_{s_0} = b_{s_0}^{f_n}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_0} = \alpha_{s_0}^{f_n}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15, 21\frac{1}{4}}$	$= \beta_{15, 28\frac{1}{3}}$
	$=$	$= \beta_{15, 21\frac{1}{4}}$	$= \beta_{15, 28\frac{1}{3}}$
$\alpha_{s_1}^{f_n} = \alpha_{s_0}^{f_n} \odot \beta_{s_0}^{l.o.f_n} = \gamma_{r_{s_1}^{f_n}, b_{s_1}^{f_n}}$	$r_{s_1}^{f_n}$	$= 5$	
	$b_{s_1}^{f_n}$	$\alpha_{s_0}^{f_n}(T_{s_0}^{l.o.f_n}) = 131\frac{1}{4}$	$\alpha_{s_0}^{f_n}(T_{s_0}^{l.o.f_n}) = 166\frac{2}{3}$
	$=$	$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$

$\text{arrivalBound}(s_1, \{f_0, f_2\}, \mathcal{G}), \mathcal{G} \in \mathcal{P}(\{f_1\}) = \alpha_{s_1}^{\{f_0, f_2\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{\{f_0, f_2\}}$		$= \gamma_{10,50}$	
$\alpha_{s_0}^{x \{f_0, f_2\}}$		$= \gamma_{0,0}$	
$\beta_{s_0}^{l.o.\{f_0, f_2\}} = \beta_{s_0} \ominus \alpha_{s_0}^{x \{f_0, f_2\}} = \beta_{R_{s_0}^{l.o.\{f_0, f_2\}}, T_{s_0}^{l.o.\{f_0, f_2\}}}$		$= \beta_{20,20}$	
$\alpha_{s_1}^{\{f_0, f_2\}} = \alpha_{s_0}^{\{f_0, f_2\}} \odot \beta_{s_0}^{l.o.\{f_0, f_2\}} = \gamma_{r_{s_1}^{\{f_0, f_2\}}, b_{s_1}^{\{f_0, f_2\}}}$	$r_{s_1}^{\{f_0, f_2\}}$	$= 10$	
	$b_{s_1}^{\{f_0, f_2\}}$	$\alpha_{s_0}^{\{f_0, f_2\}}(T_{s_0}^{l.o.\{f_0, f_2\}}) = 10 \cdot 20 + 50 = 250$	
	$=$	$= \gamma_{10,250}$	

PBOO-AB:

arrivalBound($s_2, \{f_2\}, \mathcal{G}$), $\mathcal{G} \in \mathcal{P}(\{f_1\}) = \alpha_{s_2}^{f_2}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_2}$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\alpha_{s_1}^{x(f_2)}$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\beta_{s_1}^{l.o.f_2} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_2)} = \beta_{s_1} \ominus (\alpha_{s_0}^{f_0})^* = \beta_{R_{s_1}^{l.o.f_2}, T_{s_1}^{l.o.f_2}}$	$R_{s_1}^{l.o.f_2}$	$= 15$	
	$T_{s_1}^{l.o.f_2}$	$\beta_{s_1} = b_{s_0 s_1}^{f_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{s_1} = \alpha_{s_0 s_1}^{f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{9}$
	$=$	$= \beta_{15, 26\frac{9}{16}}$	$= \beta_{15, 37\frac{7}{9}}$
	$r_{s_2}^{f_2}$	$= 5$	
$\alpha_{s_2}^{f_2} = \alpha_{s_1}^{f_2} \oslash \beta_{s_1}^{l.o.f_2} = \gamma_{r_{s_2}^{f_2}, b_{s_2}^{f_2}}$	$b_{s_2}^{f_2}$	$\alpha_{s_1}^{f_2}(T_{s_1}^{l.o.f_2}) = 5 \cdot 26\frac{9}{16} + 131\frac{1}{4} = 264\frac{1}{16}$	$\alpha_{s_1}^{f_2}(T_{s_1}^{l.o.f_2}) = 5 \cdot 37\frac{7}{9} + 166\frac{2}{3} = 355\frac{5}{9}$
	$=$	$= \gamma_{5, 264\frac{1}{16}}$	$= \gamma_{5, 355\frac{5}{9}}$

PMOO-AB, ARB_MUX:

$$\alpha_{s_2}^{f_2} = \alpha_{s_2}^{f_2} \oslash \beta_{\langle s_0, s_1 \rangle}^{l.o.f_2}$$

Note, that we use a simplified notation here due to the use of rate-latencies and token-buckets as well as the lack of demultiplexing on the analyzed path.

$$\begin{aligned}
\beta_{\langle s_0, s_1 \rangle}^{l.o.f_2} &= (\beta_{s_0} \otimes \beta_{s_1}) \ominus \alpha^{f_0} \\
&= (\beta_{20,20} \otimes \beta_{20,20}) \ominus \gamma_{5,25} \\
&= \beta_{20,40} \ominus \gamma_{5,25} \\
&= \beta_{15,55}
\end{aligned}$$

$$\begin{aligned}
\alpha_{s_2}^{f_2} &= \alpha_{s_2}^{f_2} \oslash \beta_{\langle s_0, s_1 \rangle}^{l.o.f_2} \\
&= \gamma_{5,25} \oslash \beta_{15,55} \\
&= \gamma_{5,300}
\end{aligned}$$

arrivalBound($s_2, \{f_2\}, \mathcal{G}$), $\mathcal{G} \in \mathcal{P}(\{f_1\}) = \alpha_{s_2}^{f_2}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_2}$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\alpha_{s_1}^{x(f_2)}$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\beta_{s_1}^{l.o.f_2} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_2)} = \beta_{s_1} \ominus (\alpha_{s_0}^{f_0})^* = \beta_{R_{s_1}^{l.o.f_2}, T_{s_1}^{l.o.f_2}}$	$R_{s_1}^{l.o.f_2}$	$= 15$	
	$T_{s_1}^{l.o.f_2}$	$\beta_{s_1} = b_{s_0 s_1}^{f_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{s_1} = \alpha_{s_0 s_1}^{f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{9}$
	$=$	$= \beta_{15, 26\frac{9}{16}}$	$= \beta_{15, 37\frac{7}{9}}$
	$r_{s_2}^{f_2}$	$= 5$	
$\alpha_{s_2}^{f_2} = \alpha_{s_1}^{f_2} \oslash \beta_{s_1}^{l.o.f_2} = \gamma_{r_{s_2}^{f_2}, b_{s_2}^{f_2}}$	$b_{s_2}^{f_2}$	$\alpha_{s_1}^{f_2}(T_{s_1}^{l.o.f_2}) = 5 \cdot 26\frac{9}{16} + 131\frac{1}{4} = 264\frac{1}{16}$	$\alpha_{s_1}^{f_2}(T_{s_1}^{l.o.f_2}) = 5 \cdot 37\frac{7}{9} + 166\frac{2}{3} = 355\frac{5}{9}$
	$=$	$= \gamma_{5, 264\frac{1}{16}}$	$= \gamma_{5, 355\frac{5}{9}}$

Flow f_0 (comparable to Tandem_1SC_2Flows_1AC_1Path)

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha^{f_0} + \alpha^{f_1}$	$= \gamma_{10,50}$	
	$D_{s_0}^{f_0}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{s_0}^{f_0}$	$\alpha_{s_0}(T_{s_0}) = 20 \cdot 10 + 50$ $= 250$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{\{f_0, f_2\}}$	$= \gamma_{10,250}$	
	$D_{s_1}^{f_0}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 250$ $t = 32\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 250$ $t = 65$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 250$ $= 450$	
D^{f_0}		$\sum_{i=0}^1 D_{s_i}^{f_0} = 55$	$\sum_{i=0}^1 D_{s_i}^{f_0} = 110$
B^{f_0}		$\max_{i=\{0,1\}} b_{s_i}^{f_0} = 450$	

SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)} = \alpha^{f_2}$	$= \gamma_{5,25}$	
	$\beta_{s_0}^{l.o.f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)}$	$R_{s_0}^{l.o.f_0}$	$= 5$
		$\beta_{s_0} = b_{s_0}^{x(f_0)}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_0)}$
		$20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s_1	$\alpha_{s_1}^{x(f_0)} = \alpha_{s_1}^{x(f_0)}$	$= \gamma_{5,125}$	
	$\beta_{s_1}^{l.o.f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{l.o.f_0}$	$= 15$
		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$
		$20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
	$=$	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$		$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_0} = \beta_{15,47\frac{1}{2}}$	$\bigotimes_{i=0}^1 \beta_{s_i}^{l.o.f_0} = \beta_{15,63\frac{1}{3}}$
D^{f_0}		$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 47\frac{1}{2}]^+ = 25$ $t = 49\frac{1}{6}$	$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 63\frac{1}{3}]^+ = 25$ $t = 65$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 47\frac{1}{2} + 25$ $= 262\frac{1}{2}$	$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 63\frac{1}{3} + 25$ $= 341\frac{2}{3}$

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}(f_0)}$	$= \gamma_{5,25}$
	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{5,25}$
s_1	$\alpha_{s_1}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_1}^{x(f_0)}$	$= \gamma_{5,125}$
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$	$R_{e2e}^{l.o.f_0} = \bigwedge_{i \in \{0,1\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$	$= (20 - 5) \wedge (20 - 5)$
	$T_{e2e}^{l.o.f_0} = \sum_{i \in \{0,1\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{0 + 5 \cdot 20}{15}$
	$=$	$= \beta_{15,55}$
D^{f_0}		$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 55]^+ = 25$ $t = 56\frac{2}{3}$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 55 + 25$ $= 300$

Flow f_1 (comparable with Node_2Flows_2ACs)

PBOO-AB:

TFA		FIFO_MUX	ARB_MUX
s_2	$\alpha_{s_2} = \alpha^{f_1} + \alpha_{s_1 s_2}^{f_2}$	$\gamma_{5,25} + \gamma_{5,264\frac{1}{16}} = \gamma_{10,289\frac{1}{16}}$	$\gamma_{5,25} + \gamma_{5,355\frac{5}{9}} = \gamma_{10,380\frac{5}{9}}$
	$D_{s_2}^{f_1}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t - 20]^+ = 289\frac{1}{16}$ $t = 34\frac{29}{64}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 380\frac{5}{9}$ $t = 78\frac{5}{90}$
	$B_{s_2}^{f_1}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 289\frac{1}{16}$ $= 489\frac{1}{16}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 380\frac{5}{9}$ $= 580\frac{5}{9}$
D^{f_1}		$= 34\frac{29}{64}$	$= 78\frac{5}{90}$
B^{f_1}		$= 489\frac{1}{16}$	$= 580\frac{5}{9}$

PMOO-AB:

TFA		ARB_MUX
s_2	$\alpha_{s_2} = \alpha^{f_1} + \alpha_{s_1 s_2}^{f_2}$	$\gamma_{5,25} + \gamma_{5,300} = \gamma_{10,325}$
	$D_{s_2}^{f_1}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 325$ $t = 72\frac{1}{2}$
	$B_{s_2}^{f_1}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 325$ $= 525$
D^{f_1}		$= 72\frac{1}{2}$
B^{f_1}		$= 525$

PBOO-AB:

SFA			FIFO_MUX	ARB_MUX
s2	$\alpha_{s_2}^{x(f_1)} = \alpha_{s_2}^{f_2}$		$= \gamma_{5,264 \frac{1}{16}}$	$= \gamma_{5,355 \frac{5}{9}}$
	$\beta_{s_2}^{l.o.f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)} = \beta_{s_2} \ominus \alpha_{s_1 s_2}^{x(f_1)}$	$R_{s_2}^{l.o.f_1}$	$= 15$	
		$T_{s_2}^{l.o.f_1}$	$\beta_{s_2} = b_{s_2}^{x(f_1)}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_1)}$
			$20 \cdot [t - 20]^+ = 264 \frac{1}{16}$ $t = 33 \frac{13}{64}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 355 \frac{5}{9}$ $t = 50 \frac{10}{27}$
		$=$	$= \beta_{15,33 \frac{13}{64}}$	$= \beta_{15,50 \frac{10}{27}}$
$\beta_{e2e}^{l.o.f_1} = \beta_{s_2}^{l.o.f_1}$			$= \beta_{15,33 \frac{13}{64}}$	$= \beta_{15,50 \frac{10}{27}}$
D^{f_1}			$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot [t - 33 \frac{13}{64}]^+ = 25$ $t = 34 \frac{167}{192}$	$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot [t - 50 \frac{10}{27}]^+ = 25$ $t = 52 \frac{1}{27}$
B^{f_1}			$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 33 \frac{13}{64} + 25$ $= 191 \frac{1}{64}$	$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 50 \frac{10}{27} + 25$ $= 276 \frac{23}{27}$

PMOO-AB:

SFA			ARB_MUX
s2	$\alpha_{s_2}^{x(f_1)} = \alpha_{s_2}^{f_2}$		$= \gamma_{5,300}$
	$\beta_{s_2}^{l.o.f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)} = \beta_{s_2} \ominus \alpha_{s_1 s_2}^{x(f_1)}$	$R_{s_2}^{l.o.f_1}$	$= 15$
		$T_{s_2}^{l.o.f_1}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_1)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 300$ $t = 46\frac{2}{3}$
		$=$	$= \beta_{15, 46\frac{2}{3}}$
$\beta_{e2e}^{l.o.f_1} = \beta_{s_2}^{l.o.f_1}$			$= \beta_{15, 46\frac{2}{3}}$
D^{f_1}			$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot [t - 46\frac{2}{3}]^+ = 25$ $t = 48\frac{1}{3}$
B^{f_1}			$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 46\frac{2}{3} + 25$ $= 258\frac{1}{3}$

PMOO		ARB_MUX
s_2	$\alpha_{s_2}^{\bar{x}(f_1)}$	$= \gamma_{5,355\frac{5}{9}}$
	$\alpha_{s_2}^{x(f_1)}$	$= \gamma_{5,355\frac{5}{9}}$
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$	$R_{e2e}^{l.o.f_1} = R_{s_2} - r_{s_2}^{x(f_0)}$	$= 20 - 5$
		$= 15$
	$T_{e2e}^{l.o.f_1} = T_{s_2} + \frac{b_{s_2}^{\bar{x}(f_1)} + r_{s_2}^{x(f_1)} \cdot T_{s_2}}{R_{e2e}^{l.o.f_1}}$	$= 20 + \frac{355\frac{5}{9} + 5 \cdot 20}{15}$
		$= 50\frac{10}{27}$
	$=$	$= \beta_{15,50\frac{10}{27}}$
D^{f_1}		$\beta_{e2e}^{l.o.f_1} = b^{f_1}$
		$15 \cdot [t - 50\frac{10}{27}]^+ = 25$
		$t = 52\frac{1}{27}$
B^{f_1}		$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 50\frac{10}{27} + 25$
		$= 276\frac{23}{27}$

Flow f_2

PBOO-AB:

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_1}$	$= \gamma_{10,50}$	
	$D_{s_0}^{f_2}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{s_0}^{f_2}$	$\alpha_{s_0}(T_{s_0}) = 20 \cdot 10 + 50$ $= 250$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{\{f_0, f_1\}}$	$= \gamma_{10,250}$	
	$D_{s_1}^{f_2}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 250$ $t = 32\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 250$ $t = 65$
	$B_{s_1}^{f_2}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 250$ $= 450$	
s_2	$\alpha_{s_2} = \alpha_{s_2}^{f_1} + \alpha_{s_2}^{f_2}$	$\gamma_{5,25} + \gamma_{5,264\frac{1}{16}} = \gamma_{10,289\frac{1}{16}}$	$\gamma_{5,25} + \gamma_{5,355\frac{5}{9}} = \gamma_{10,380\frac{5}{9}}$
	$D_{s_2}^{f_2}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t - 20]^+ = 289\frac{1}{16}$ $t = 34\frac{29}{64}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 380\frac{5}{9}$ $t = 78\frac{5}{90}$
	$B_{s_2}^{f_2}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 289\frac{1}{16}$ $= 489\frac{1}{16}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 380\frac{5}{9}$ $= 580\frac{5}{9}$
D^{f_2}		$\sum_{i=0}^2 D_{s_i}^{f_2} = 89\frac{29}{64}$	$\sum_{i=0}^2 D_{s_i}^{f_2} = 188\frac{5}{90}$
B^{f_2}		$\max_{i=\{0,1,2\}} b_{s_i}^{f_0} = 489\frac{1}{16}$	$\max_{i=\{0,1,2\}} b_{s_i}^{f_0} = 580\frac{5}{9}$

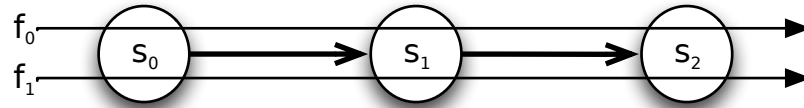
PMOO-AB:

TFA		ARB_MUX
s_0	$\alpha_{s_0} = \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_1}$	$= \gamma_{10,50}$
	$D_{s_0}^{f_2}$	$\beta_{s_0} = \alpha_{s_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{s_0}^{f_2}$	$\alpha_{s_0}(T_{s_0}) = 20 \cdot 10 + 50$ $= 250$
s_1	$\alpha_{s_1} = \alpha_{s_1}^{\{f_0, f_1\}}$	$= \gamma_{10,250}$
	$D_{s_1}^{f_2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 250$ $t = 65$
	$B_{s_1}^{f_2}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 250$ $= 450$
s_2	$\alpha_{s_2} = \alpha_{s_1}^{f_1} + \alpha_{s_1 s_2}^{f_2}$	$\gamma_{5,25} + \gamma_{5,300} = \gamma_{10,325}$
	$D_{s_2}^{f_1}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 325$ $t = 72\frac{1}{2}$
	$B_{s_2}^{f_1}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 325$ $= 525$
D^{f_2}		$\sum_{i=0}^2 D_{s_i}^{f_2} = 182\frac{1}{2}$
B^{f_2}		$\max_{i=\{0,1,2\}} b_{s_i}^{f_0} = 525$

SFA			FIFO_MUX	ARB_MUX
s ₀	$\alpha_{s_0}^{x(f_2)} = \alpha_{s_0}^{f_0}$		$= \gamma_{5,25}$	
	$\beta_{s_0}^{l.o.f_2} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_2)} = \beta_{s_0} \ominus \alpha_{s_0}^{f_0}$	$R_{s_0}^{l.o.f_2}$	$= 5$	
		$T_{s_0}^{l.o.f_2}$	$\beta_{s_0} = b_{s_0}^{x(f_2)}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_0} = \alpha_{s_0}^{x(f_2)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
		$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s ₁	$\alpha_{s_1}^{x(f_2)} = \alpha_{s_1}^{x(f_2)}$		$= \gamma_{5,125}$	
	$\beta_{s_1}^{l.o.f_2} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_2)}$	$R_{s_1}^{l.o.f_2}$	$= 15$	
		$T_{s_1}^{l.o.f_2}$	$\beta_{s_1} = b_{s_1}^{x(f_2)}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_2)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
		$=$	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$
s ₂	$\alpha_{s_2}^{x(f_2)} = \alpha_{s_2}^{f_1}$		$= \gamma_{5,25}$	
	$\beta_{s_2}^{l.o.f_2} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_2)} = \beta_{s_2} \ominus \alpha_{s_2}^{f_1}$	$R_{s_2}^{l.o.f_2}$	$= 15$	
		$T_{s_2}^{l.o.f_2}$	$\beta_{s_2} = b_{s_2}^{x(f_2)}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_2)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
		$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
$\beta_{e2e}^{l.o.f_2}$			$\bigotimes_{i=0}^2 \beta_{s_i}^{l.o.f_2} = \beta_{15,68\frac{3}{4}}$	$\bigotimes_{i=0}^2 \beta_{s_i}^{l.o.f_0} = \beta_{15,91\frac{2}{3}}$
D^{f_2}			$\beta_{e2e}^{l.o.f_2} = b^{f_2}$ $15 \cdot [t - 68\frac{3}{4}]^+ = 25$ $t = 70\frac{5}{12}$	$\beta_{e2e}^{l.o.f_2} = b^{f_2}$ $15 \cdot [t - 91\frac{2}{3}]^+ = 25$ $t = 93\frac{1}{3}$
B^{f_2}			$\alpha^{f_2}(T_{s_2}^{l.o.f_2}) = 5 \cdot 68\frac{3}{4} + 25$ $= 368\frac{3}{4}$	$\alpha^{f_2}(T_{e2e}^{l.o.f_2}) = 5 \cdot 91\frac{2}{3} + 25$ $= 483\frac{1}{3}$

PMOO		ARB_MUX
s_0	$\frac{\bar{x}(f_2)}{\alpha_{s_0}}$	$= \gamma_{5,25}$
	$\frac{x(f_2)}{\alpha_{s_0}}$	$= \gamma_{5,25}$
s_1	$\frac{\bar{x}(f_2)}{\alpha_{s_1}}$	$= \gamma_{0,0}$
	$\frac{x(f_2)}{\alpha_{s_1}}$	$= \gamma_{5,125}$
s_2	$\frac{\bar{x}(f_2)}{\alpha_{s_2}}$	$= \gamma_{5,25}$
	$\frac{x(f_2)}{\alpha_{s_2}}$	$= \gamma_{5,25}$
$\beta_{e2e}^{l.o.f_2} = \beta_{R_{e2e}^{l.o.f_2}, T_{e2e}^{l.o.f_2}}$	$R_{e2e}^{l.o.f_2} = \bigwedge_{i \in \{0,1,2\}} \left(R_{s_i} - r_{s_i}^{x(f_2)} \right)$	$= (20 - 5) \wedge (20 - 5) \wedge (20 - 5)$
	$T_{e2e}^{l.o.f_2} = \sum_{i \in \{0,1,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_2)} + r_{s_i}^{x(f_2)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{0 + 5 \cdot 20}{15} + 20 + \frac{25 + 5 \cdot 20}{15}$
	$=$	$= 83\frac{1}{3}$
D^{f_2}		$\beta_{e2e}^{l.o.f_2} = b^{f_1}$
		$15 \cdot [t - 83\frac{1}{3}] = 25$
		$t = 85$
B^{f_2}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 83\frac{1}{3} + 25$
		$= 441\frac{2}{3}$

Tandem_1SC_2Flows_1AC_1Path_v2



- $\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0,1\}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0,1\}$

arrivalBound($s_1, \{f_0, f_1\}, \{\}\rangle = \alpha_{s_1}^{\{f_0, f_1\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{\{f_0, f_1\}}$			$= \gamma_{10,50}$
$\alpha_{s_0}^{x\{f_0, f_1\}}$			$= \gamma_{0,0}$
$\beta_{s_0}^{l.o.\{f_0, f_1\}} = \beta_{s_0} \ominus \alpha_{s_0}^{x\{f_0, f_1\}} = \beta_{R_{s_0}^{l.o.\{f_0, f_1\}}, T_{s_0}^{l.o.\{f_0, f_1\}}}$			$= \beta_{20,20}$
$\alpha_{s_1}^{\{f_0, f_1\}} = \alpha_{s_0}^{\{f_0, f_1\}} \oslash \beta_{s_0}^{l.o.\{f_0, f_1\}} = \gamma_{r_{s_1}^{\{f_0, f_1\}}, b_{s_1}^{\{f_0, f_1\}}}$	$r_{s_1}^{\{f_0, f_1\}}$		$= 10$
	$b_{s_1}^{\{f_0, f_1\}}$	$\alpha_{s_0}^{\{f_0, f_1\}}(T_{s_0}^{l.o.\{f_0, f_1\}}) = 10 \cdot 20 + 50 = 250$	
	$=$		$= \gamma_{10,250}$

arrivalBound($s_2, \{f_0, f_1\}, \{\}\rangle = \alpha_{s_2}^{\{f_0, f_1\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{\{f_0, f_1\}}$			$= \gamma_{10,250}$
$\alpha_{s_1}^{x\{f_0, f_1\}}$			$= \gamma_{0,0}$
$\beta_{s_1}^{l.o.\{f_0, f_1\}} = \beta_{s_1} \ominus \alpha_{s_1}^{x\{f_0, f_1\}} = \beta_{R_{s_1}^{l.o.\{f_0, f_1\}}, T_{s_1}^{l.o.\{f_0, f_1\}}}$			$= \beta_{20,20}$
$\alpha_{s_2}^{\{f_0, f_1\}} = \alpha_{s_1}^{\{f_0, f_1\}} \oslash \beta_{s_1}^{l.o.\{f_0, f_1\}} = \gamma_{r_{s_2}^{\{f_0, f_1\}}, b_{s_2}^{\{f_0, f_1\}}}$	$r_{s_2}^{\{f_0, f_1\}}$		$= 10$
	$b_{s_2}^{\{f_0, f_1\}}$	$\alpha_{s_1}^{\{f_0, f_1\}}(T_{s_1}^{l.o.\{f_0, f_1\}}) = 10 \cdot 20 + 250 = 250$	
	$=$		$= \gamma_{10,450}$

arrivalBound($s_1, \{f_0\}, \{f_1\}\rangle = \alpha_{s_1}^{f_0}$ = arrivalBound($s_1, \{f_1\}, \{f_0\}\rangle = \alpha_{s_1}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_n}$			$= \gamma_{5,25}$
$\alpha_{s_0}^{x f_n}$			$= \gamma_{0,0}$
$\beta_{s_0}^{l.o.f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{x f_n} = \beta_{R_{s_0}^{l.o.f_n}, T_{s_0}^{l.o.f_n}}$			$= \beta_{20,20}$
$\alpha_{s_1}^{f_n} = \alpha_{s_0}^{f_n} \oslash \beta_{s_0}^{l.o.f_n} = \gamma_{r_{s_1}^{f_n}, b_{s_1}^{f_n}}$	$r_{s_1}^{f_n}$		$= 5$
	$b_{s_1}^{f_n}$	$\alpha_{s_0}^{f_n}(T_{s_0}^{l.o.f_n}) = 5 \cdot 20 + 25 = 125$	
	$=$		$= \gamma_{5,125}$

$\text{arrivalBound}(s_2, \{f_0\}, \{f_1\}) = \alpha_{s_2}^{f_0}$ $= \text{arrivalBound}(s_2, \{f_1\}, \{f_0\}) = \alpha_{s_2}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_n}$		$= \gamma_{5,125}$	
$\alpha_{s_1}^{x f_n}$		$= \gamma_{0,0}$	
$\beta_{s_1}^{\text{l.o.} f_n} = \beta_{s_1} \ominus \alpha_{s_1}^{x f_n} = \beta_{R_{s_1}^{\text{l.o.} f_n}, T_{s_1}^{\text{l.o.} f_n}}$		$= \beta_{20,20}$	
$\alpha_{s_2}^{f_n} = \alpha_{s_1}^{f_n} \oslash \beta_{s_1}^{\text{l.o.} f_n} = \gamma_{r_{s_2}^{f_n}, b_{s_2}^{f_n}}$	$r_{s_2}^{f_n}$	$= 5$	
	$b_{s_2}^{f_n}$	$\alpha_{s_1}^{f_n}(T_{s_1}^{\text{l.o.} f_n}) = 5 \cdot 20 + 125 = 225$	
	$=$	$= \gamma_{5,125}$	

Flows $f_n, n \in \{0, 1\}$

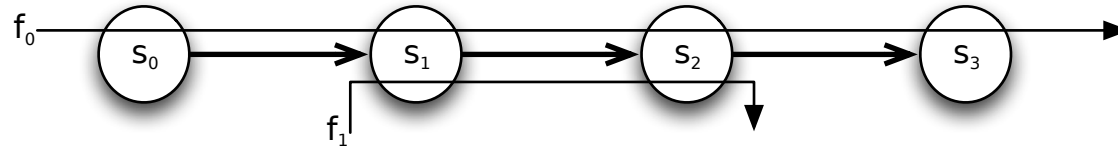
TFA results will be equal for all flows as they share the same path of servers.

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0} = \alpha_{s_0}^{f_0} + \alpha_{s_0}^{f_1}$	$= \gamma_{10,50}$	
	$D_{s_0}^{f_n}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{s_0} = \alpha_{s_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{s_0}^{f_n}$	$\alpha_{s_0}(T_{s_0}) = 10 \cdot 20 + 50$ $= 250$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{\{f_0, f_1\}}$	$= \gamma_{10,250}$	
	$D_{s_1}^{f_n}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 250$ $t = 32\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 250$ $t = 65$
	$B_{s_1}^{f_n}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 250$ $= 450$	
s_2	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$	$= \gamma_{10,450}$	
	$D_{s_2}^{f_n}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t - 20]^+ = 450$ $t = 42\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 450$ $t = 85$
	$B_{s_2}^{f_n}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 450$ $= 650$	
$D_{s_i}^{f_n}$		$\sum_{i=0}^2 D_{s_i}^{f_n} = 97\frac{1}{2}$	$\sum_{i=0}^2 D_{s_i}^{f_n} = 195$
$B_{s_i}^{f_n}$		$\max_{i=\{0,1,2\}} b_{s_i}^{f_n} = 650$	

SFA			FIFO_MUX	ARB_MUX
s0	$\alpha_{s_0}^{xf_n}$		$= \gamma_{5,25}$	
	$\beta_{s_0}^{l.o.f_n} = \beta_{s_0} \ominus \alpha_{s_0}^{xf_n} = \beta_{R_{s_0}^{l.o.f_n}, T_{s_0}^{l.o.f_n}}$	$R_{s_0}^{l.o.f_n}$	$= 15$	
		$T_{s_0}^{l.o.f_n}$	$\beta_{s_0} = b_{s_0}^{xf_n}$ $20 \cdot [t - 20]^+ = 25$ $t = 22\frac{1}{4}$	$\beta_{s_0} = \alpha_{s_0}^{xf_n}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
		$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s1	$\alpha_{s_1}^{xf_n}$		$= \gamma_{5,125}$	
	$\beta_{s_1}^{l.o.f_n} = \beta_{s_1} \ominus \alpha_{s_1}^{xf_n}$	$R_{s_1}^{l.o.f_n}$	$= 15$	
		$T_{s_1}^{l.o.f_n}$	$\beta_{s_1} = b_{s_1}^{xf_n}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$\beta_{s_1} = \alpha_{s_1}^{xf_n}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
		$=$	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$
s2	$\alpha_{s_2}^{xf_n}$		$= \gamma_{5,225}$	
	$\beta_{s_2}^{l.o.f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{xf_n}$	$R_{s_2}^{l.o.f_n}$	$= 15$	
		$T_{s_2}^{l.o.f_n}$	$\beta_{s_2} = b_{s_2}^{xf_n}$ $20 \cdot [t - 20]^+ = 225$ $t = 31\frac{1}{4}$	$\beta_{s_2} = \alpha_{s_2}^{xf_n}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 225$ $t = 41\frac{2}{3}$
		$=$	$= \beta_{15,31\frac{1}{4}}$	$= \beta_{15,41\frac{2}{3}}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$			$\bigotimes_{i=0}^2 \beta_{s_i}^{l.o.f_n} = \beta_{5,78\frac{3}{4}}$	$\bigotimes_{i=0}^2 \beta_{s_i}^{l.o.f_n} = \beta_{5,105}$
D^{f_n}			$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $15 \cdot [t - 78\frac{3}{4}]^+ = 25$ $t = 80\frac{5}{12}$	$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $15 \cdot [t - 105]^+ = 25$ $t = 106\frac{2}{3}$
B^{f_n}			$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 78\frac{3}{4} + 25$ $= 418\frac{3}{4}$	$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 105 + 25$ $= 550$

PMOO		ARB_MUX
s_0	$\alpha_{s_0}^{\bar{x}f_n}$	$= \gamma_{5,25}$
	$\alpha_{s_0}^{xf_n}$	$= \gamma_{5,25}$
s_1	$\alpha_{s_1}^{\bar{x}f_n}$	$= \gamma_{0,0}$
	$\alpha_{s_1}^{xf_n}$	$= \gamma_{5,75}$
s_2	$\alpha_{s_2}^{\bar{x}f_n}$	$= \gamma_{0,0}$
	$\alpha_{s_2}^{xf_n}$	$= \gamma_{5,225}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$	$R_{e2e}^{l.o.f_n} = \bigwedge_{i \in \{0,1,2\}} (R_{s_i} - r_{s_i}^{xf_n})$	$= (20 - 5) \wedge (20 - 5) \wedge (20 - 5)$
	$T_{e2e}^{l.o.f_n} = \sum_{i \in \{0,1,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}f_n} + r_{s_i}^{xf_n} \cdot T_{s_i}}{R_{e2e}^{l.o.f_n}} \right)$	$= 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{0 + 5 \cdot 20}{15} + 20 + \frac{0 + 5 \cdot 20}{15}$
	$=$	$= 81\frac{2}{3}$
D^{f_n}		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $15 \cdot [t - 81\frac{2}{3}]^+ = 25$ $t = 83\frac{1}{3}$
B^{f_n}		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 81\frac{2}{3} + 25$ $= 433\frac{1}{3}$

Tandem_1SC_2Flows_1AC_2Paths_v2



- $\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{s_3} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0,1\}$
- $\mathcal{F} = \{f_0, f_1\}$

$$\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0,1\}$$

arrivalBound($s_1, \{f_0\}, \mathcal{G}$), $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_0}$			$= \gamma_{5,25}$
$\alpha_{s_0}^{x(f_0)}$			$= \gamma_{0,0}$
$\beta_{s_0}^{l.o.f_0} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_0)} = \beta_{R_{s_0}^{l.o.f_0}, T_{s_0}^{l.o.f_0}}$			$= \beta_{20,20}$
$\alpha_{s_1}^{f_0} = \alpha_{s_0}^{f_0} \oslash \beta_{s_0}^{l.o.f_0} = \gamma_{r_{s_1}^{f_0}, b_{s_1}^{f_0}}$	$r_{s_1}^{f_0}$		$= 10$
	$b_{s_1}^{f_0}$	$\alpha_{s_0}^{f_0}(T_{s_0}^{l.o.f_0}) = 5 \cdot 20 + 25 = 125$	
	$=$		$= \gamma_{5,125}$

arrivalBound($s_2, \{f_0\}, \{f_0\}) = \alpha_{s_2}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_0}$			$= \gamma_{5,125}$
$\alpha_{s_1}^{x(f_0)}$			$= \gamma_{5,25}$
$\beta_{s_1}^{l.o.f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{l.o.f_0}$		$= 15$
	$T_{s_1}^{l.o.f_0}$	$\beta_{s_1} = b_{s_1}^{x(f_0)}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15, 21\frac{1}{4}}$	$= \beta_{15, 28\frac{1}{3}}$
	$r_{s_2}^{f_0}$	$= 5$	
$\alpha_{s_2}^{f_0} = \alpha_{s_1}^{f_0} \oslash \beta_{s_1}^{l.o.f_0} = \gamma_{r_{s_2}^{f_0}, b_{s_2}^{f_0}}$	$b_{s_2}^{f_0}$	$\alpha_{s_1}^{f_0}(T_{s_1}^{l.o.f_0}) = 5 \cdot 21\frac{1}{4} + 125 = 231\frac{1}{4}$	$\alpha_{s_1}^{f_0}(T_{s_1}^{l.o.f_0}) = 5 \cdot 28\frac{1}{3} + 125 = 266\frac{2}{3}$
	$=$	$= \gamma_{5, 231\frac{1}{4}}$	$= \gamma_{5, 266\frac{2}{3}}$

arrivalBound($s_2, \{f_1\}, \{f_0\}) = \alpha_{s_2}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_1}$			$= \gamma_{5,25}$
$\alpha_{s_1}^{x(f_1)}$			$= \gamma_{0,0}$
$\beta_{s_1}^{l.o.f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)} = \beta_{R_{s_1}^{l.o.f_1}, T_{s_1}^{l.o.f_1}}$			$= \beta_{20,20}$
$\alpha_{s_2}^{f_1} = \alpha_{s_1}^{f_1} \oslash \beta_{s_1}^{l.o.f_1} = \gamma_{r_{s_2}^{f_1}, b_{s_2}^{f_1}}$	$r_{s_2}^{f_1}$		$= 10$
	$b_{s_2}^{f_1}$	$\alpha_{s_1}^{f_1}(T_{s_1}^{l.o.f_1}) = 5 \cdot 20 + 25 = 125$	
	$=$		$= \gamma_{5,125}$

PBOO-AB:

arrivalBound($s_3, \{f_0\}, \{\}$) = $\alpha_{s_3}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{s_2}^{f_0}$		$= \gamma_{5,231\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}}$
$\alpha_{s_2}^{x(f_0)}$		$= \gamma_{5,125}$	
$\beta_{s_2}^{l.o.f_0} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_0)}$	$R_{s_2}^{l.o.f_0}$	$= 15$	
	$T_{s_2}^{l.o.f_0}$	$\beta_{s_2} = b_{s_2}^{x(f_0)}$ $20 \cdot [t - 20]^+ = 156\frac{1}{4}$ $t = 27\frac{13}{16}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_0)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 200$ $t = 40$
	$=$	$= \beta_{15,27\frac{13}{16}}$	$= \beta_{15,40}$
	$r_{s_3}^{f_0}$	$= 5$	
$\alpha_{s_3}^{f_0} = \alpha_{s_2}^{f_0} \oslash \beta_{s_2}^{l.o.f_0} = \gamma_{r_{s_3}^{f_1}, b_{s_3}^{f_1}}$	$b_{s_3}^{f_0}$	$\alpha_{s_2}^{f_0}(T_{s_2}^{l.o.f_0}) = 370\frac{5}{16}$	$\alpha_{s_2}^{f_0}(T_{s_2}^{l.o.f_0}) = 466\frac{2}{3}$
	$=$	$= \gamma_{5,370\frac{5}{16}}$	$= \gamma_{5,466\frac{2}{3}}$

PMOO-AB, ARB_MUX:

$$\alpha_{s_3}^{f_0} = \alpha^{f_0} \oslash \beta_{\langle s_0, s_2 \rangle}^{l.o.f_0}$$

Note, that we use a simplified notation here due to the use of rate-latencies and token-buckets as well as the lack of demultiplexing on the analyzed path.

$$\begin{aligned}
\beta_{\langle s_0, s_2 \rangle}^{l.o.f_0} &= \beta_{s_0} \otimes ((\beta_{s_1} \otimes \beta_{s_2}) \ominus \alpha^{f_1}) \\
&= \beta_{20,20} \otimes ((\beta_{20,20} \otimes \beta_{20,20}) \ominus \gamma_{5,25}) \\
&= \beta_{20,20} \otimes (\beta_{20,40} \ominus \gamma_{5,25}) \\
&= \beta_{20,20} \otimes \beta_{15,55} \\
&= \beta_{15,75}
\end{aligned}$$

$$\begin{aligned}
\alpha_{s_3}^{f_0} &= \alpha^{f_0} \oslash \beta_{\langle s_0, s_2 \rangle}^{l.o.f_0} \\
&= \gamma_{5,25} \oslash \beta_{15,75} \\
&= \gamma_{5,400}
\end{aligned}$$

$\text{arrivalBound}(s_2, \{f_0, f_1\}, \{\}) = \alpha_{s_2}^{\{f_0, f_1\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{\{f_0, f_1\}}$			$= \gamma_{10,150}$
$\alpha_{s_1}^{x\{f_0, f_1\}}$			$= \gamma_{0,0}$
$\beta_{s_1}^{1.o.\{f_0, f_1\}} = \beta_{s_1} \ominus \alpha_{s_1}^{x\{f_0, f_1\}} = \beta_{R_{s_1}^{1.o.\{f_0, f_1\}}, T_{s_1}^{1.o.\{f_0, f_1\}}}$			$= \beta_{20,20}$
$\alpha_{s_2}^{\{f_0, f_1\}} = \alpha_{s_1}^{\{f_0, f_1\}} \oslash \beta_{s_1}^{1.o.\{f_0, f_1\}} = \gamma_{r_{s_2}^{\{f_0, f_1\}}, b_{s_2}^{\{f_0, f_1\}}}$	$r_{s_2}^{\{f_0, f_1\}}$		$= 10$
	$b_{s_2}^{\{f_0, f_1\}}$	$\alpha_{s_1}^{\{f_0, f_1\}}(T_{s_1}^{1.o.\{f_0, f_1\}})$	$= 10 \cdot 20 + 150 = 350$
	$=$		$= \gamma_{10,350}$

$\text{arrivalBound}(s_2, \{f_0\}, \{f_1\}) = \alpha_{s_2}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_0}$			$= \gamma_{5,125}$
$\alpha_{s_1}^{x(f_0)}$			$= \gamma_{0,0}$
$\beta_{s_1}^{1.o.f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)} = \beta_{R_{s_1}^{1.o.f_0}, T_{s_1}^{1.o.f_0}}$			$= \beta_{20,20}$
$\alpha_{s_2}^{f_0} = \alpha_{s_1}^{f_0} \oslash \beta_{s_1}^{1.o.f_0} = \gamma_{r_{s_2}^{f_0}, b_{s_2}^{f_0}}$	$r_{s_2}^{f_0}$		$= 10$
	$b_{s_2}^{f_0}$	$\alpha_{s_1}^{f_0}(T_{s_1}^{1.o.f_0})$	$= 5 \cdot 20 + 125 = 225$
	$=$		$= \gamma_{5,225}$

Flow f_0

PBOO-AB:

	TFA	FIFO_MUX	ARB_MUX
s_0	α_{s_0}	$= \gamma_{5,25}$	
	$D_{s_0}^{f_0}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per microflow $\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_0}^{f_0}$	$\alpha_{s_0}(T_{s_0}) = 5 \cdot 20 + 25$ $= 125$	
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_1}$	$= \gamma_{5,125} + \gamma_{5,25} = \gamma_{10,150}$	
	$D_{s_1}^{f_0}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 150$ $t = 27\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 150$ $t = 55$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 150$ $= 350$	
s_2	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$	$= \gamma_{10,350}$	
	$D_{s_2}^{f_0}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t - 20]^+ = 350$ $t = 37\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 350$ $t = 75$
	$B_{s_2}^{f_0}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 350$ $= 550$	
s_3	$\alpha_{s_3} = \alpha_{s_3}^{f_0}$	$= \gamma_{5,370\frac{5}{16}}$	$= \gamma_{5,466\frac{2}{3}}$
	$D_{s_3}^{f_0}$	$\beta_{s_3} = b_{s_3}$ $20 \cdot [t - 20]^+ = 370\frac{5}{16}$ $t = 38\frac{33}{64}$	FIFO per micro flow $\beta_{s_3} = b_{s_3}$ $20 \cdot [t - 20]^+ = 466\frac{2}{3}$ $t = 43\frac{1}{3}$
	$B_{s_3}^{f_0}$	$\alpha_{s_3}(T_{s_3}) = 5 \cdot 20 + 370\frac{5}{16}$ $= 470\frac{5}{16}$	$\alpha_{s_3}(T_{s_3}) = 5 \cdot 20 + 466\frac{2}{3}$ $= 566\frac{2}{3}$
D^{f_0}		$\sum_{i=0}^3 D_{s_i}^{f_0} = 124\frac{49}{64}$	$\sum_{i=0}^3 D_{s_i}^{f_0} = 194\frac{7}{12}$
B^{f_0}		$\max_{i=0}^3 b_{s_i}^{f_0} = 550$	$\max_{i=0}^3 b_{s_i}^{f_0} = 566\frac{2}{3}$

PMOO-AB:

TFA		ARB_MUX
s_0	α_{s_0}	$= \gamma_{5,25}$
	$D_{s_0}^{f_0}$	FIFO per microflow $\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_0}^{f_0}$	$\alpha_{s_0}(T_{s_0}) = 125$
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_1}$	$= \gamma_{5,125} + \gamma_{5,25} = \gamma_{10,150}$
	$D_{s_1}^{f_0}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 150$ $t = 55$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 150$ $= 350$
s_2	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$	$= \gamma_{10,350}$
	$D_{s_2}^{f_0}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 350$ $t = 75$
	$B_{s_2}^{f_0}$	$\alpha_{s_2}(T_{s_2}) = 550$
s_3	$\alpha_{s_3} = \alpha_{s_3}^{f_0}$	$= \gamma_{5,400}$
	$D_{s_3}^{f_0}$	FIFO per micro flow $\beta_{s_3} = b_{s_3}$ $20 \cdot [t - 20]^+ = 400$ $t = 40$
	$B_{s_3}^{f_0}$	$\alpha_{s_3}(T_{s_3}) = 5 \cdot 20 + 400$ $= 500$
D^{f_0}		$\sum_{i=0}^3 D_{s_i}^{f_0} = 191\frac{1}{4}$
B^{f_0}		$\max_{i=0}^3 b_{s_i}^{f_0} = 550$

SFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_0}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{s_0}^{l.o.f_0}$	$= \beta_{20,20}$	
s_1	$\alpha_{s_1}^{x(f_0)}$	$= \gamma_{5,25}$	
	$\beta_{s_1}^{l.o.f_0} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_0)}$	$R_{s_1}^{l.o.f_0}$	$= 15$
		$\beta_{s_1} = b_{s_1}^{x(f_0)}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$
		$20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
s_2	$\alpha_{s_2}^{x(f_0)}$	$= \gamma_{5,125}$	
	$\beta_{s_2}^{l.o.f_0} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_0)}$	$R_{s_2}^{l.o.f_0}$	$= 15$
		$\beta_{s_2} = b_{s_2}^{x(f_0)}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_0)}$
		$20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
	$=$	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$
s_3	$\alpha_{s_3}^{x(f_0)}$	$= \gamma_{0,0}$	
	$\beta_{s_3}^{l.o.f_0} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_0)}$	$= \beta_{20,20}$	
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$		$\bigotimes_{i=0}^3 \beta_{s_i}^{l.o.f_0} = \beta_{15,87\frac{1}{2}}$	$\bigotimes_{i=0}^3 \beta_{s_i}^{l.o.f_0} = \beta_{15,103\frac{1}{3}}$
D^{f_0}		$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 87\frac{1}{2}]^+ = 25$ $t = 89\frac{1}{6}$	$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 103\frac{1}{3}]^+ = 25$ $t = 105$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 87\frac{1}{2} + 25$ $= 462\frac{1}{2}$	$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 103\frac{1}{3} + 25$ $= 541\frac{2}{3}$

PMOO		ARB_MUX
s_0	$\frac{\bar{x}(f_0)}{\alpha_{s_0}}$	$= \gamma_{0,0}$
	$\frac{x(f_0)}{\alpha_{s_0}}$	$= \gamma_{0,0}$
s_1	$\frac{\bar{x}(f_0)}{\alpha_{s_1}}$	$= \gamma_{5,25}$
	$\frac{x(f_0)}{\alpha_{s_1}}$	$= \gamma_{5,25}$
s_2	$\frac{\bar{x}(f_0)}{\alpha_{s_2}}$	$= \gamma_{0,0}$
	$\frac{x(f_0)}{\alpha_{s_2}}$	$= \gamma_{5,125}$
s_3	$\frac{\bar{x}(f_0)}{\alpha_{s_3}}$	$= \gamma_{0,0}$
	$\frac{x(f_0)}{\alpha_{s_3}}$	$= \gamma_{0,0}$
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$	$R_{e2e}^{l.o.f_0} = \bigwedge_{i \in \{0,1,2,3\}} \left(R_{s_i} - r_{s_i}^{x(f_0)} \right)$	$= (20 - 0) \wedge (20 - 5) \wedge (20 - 5) \wedge (20 - 0)$ $= 15$
	$T_{e2e}^{l.o.f_0} = \sum_{i \in \{0,1,2,3\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 20 + \frac{0 + 0 \cdot 20}{15} + 20 + \frac{25 + 5 \cdot 20}{15} + 20 + \frac{0 + 5 \cdot 20}{15} + 20 + \frac{0 + 0 \cdot 20}{15}$ $= 95$
	$=$	$= \beta_{15,95}$
D^{f_0}		$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 95]^+ = 25$ $t = 96\frac{2}{3}$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 95 + 25$ $= 500$

Flow f_1

TFA		FIFO_MUX	ARB_MUX
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_1}$	$= \gamma_{5,25} + \gamma_{5,125} = \gamma_{10,150}$	
	$D_{s_1}^{f_1}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 150$ $t = 27\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 150$ $t = 55$
	$B_{s_1}^{f_1}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 150$ $= 350$	
s_2	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$	$= \gamma_{10,350}$	
	$D_{s_2}^{f_1}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t - 20]^+ = 350$ $t = 37\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 350$ $t = 75$
	$B_{s_2}^{f_1}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 350$ $= 550$	
D^{f_1}		$\sum_{i=1}^2 D_{s_i}^{f_1} = 65$	$\sum_{i=1}^2 D_{s_i}^{f_1} = 130$
B^{f_1}		$\max_{i=1}^2 b_{s_i}^{f_1} = 550$	

SFA			FIFO_MUX	ARB_MUX
s ₁	$\alpha_{s_1}^{x(f_1)}$		$= \gamma_{5,125}$	
	$\beta_{s_1}^{l.o.f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)}$	$R_{s_1}^{l.o.f_1}$	$= 15$	
		$T_{s_1}^{l.o.f_1}$	$\beta_{s_1} = b_{s_1}^{x(f_1)}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$\beta_{s_1} = \alpha_{s_1}^{x(f_0)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
		$=$	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$
s ₂	$\alpha_{s_2}^{x(f_1)}$		$= \gamma_{5,225}$	
	$\beta_{s_2}^{l.o.f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)}$	$R_{s_2}^{l.o.f_1}$	$= 15$	
		$T_{s_2}^{l.o.f_1}$	$\beta_{s_2} = b_{s_2}^{x(f_1)}$ $20 \cdot [t - 20]^+ = 225$ $t = 31\frac{1}{4}$	$\beta_{s_2} = \alpha_{s_2}^{x(f_1)}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 225$ $t = 41\frac{2}{3}$
		$=$	$= \beta_{15,31\frac{1}{4}}$	$= \beta_{15,41\frac{2}{3}}$
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$		$\bigotimes_{i=1}^2 \beta_{s_i}^{l.o.f_1} = \beta_{15,57\frac{1}{2}}$	$\bigotimes_{i=1}^2 \beta_{s_i}^{l.o.f_1} = \beta_{15,76\frac{2}{3}}$	
D^{f_1}		$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot [t - 57\frac{1}{2}]^+ = 25$ $t = 59\frac{1}{6}$	$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $15 \cdot [t - 76\frac{2}{3}]^+ = 25$ $t = 78\frac{1}{3}$	
B^{f_1}		$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 57\frac{1}{2} + 25$ $= 312\frac{1}{2}$	$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 76\frac{2}{3} + 25$ $= 408\frac{1}{3}$	

PMOO		ARB_MUX
s_1	$\alpha_{s_1}^{\bar{x}(f_1)}$	$= \gamma_{5,125}$
	$\alpha_{s_1}^{x(f_1)}$	$= \gamma_{5,125}$
s_2	$\alpha_{s_2}^{\bar{x}(f_1)}$	$= \gamma_{0,0}$
	$\alpha_{s_2}^{x(f_1)}$	$= \gamma_{5,225}$
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$	$R_{e2e}^{l.o.f_1} = \bigwedge_{i \in \{1,2\}} \left(R_{s_i} - r_{s_i}^{x(f_1)} \right)$	$= (20 - 5) \wedge (20 - 5)$
	$T_{e2e}^{l.o.f_1} = \sum_{i \in \{1,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_1}} \right)$	$= 20 + \frac{125 + 5 \cdot 20}{15} + 20 + \frac{0 + 5 \cdot 20}{15}$
	$=$	$= 61\frac{2}{3}$
D^{f_1}		$\beta_{e2e}^{l.o.f_1} = b^{f_1}$
		$15 \cdot [t - 61\frac{2}{3}]^+ = 25$
		$t = 63\frac{1}{3}$
B^{f_1}		$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 61\frac{2}{3} + 25$
		$= 333\frac{1}{3}$