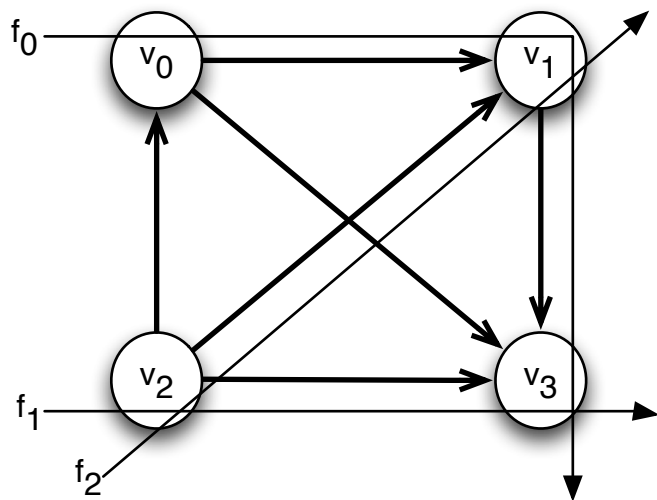
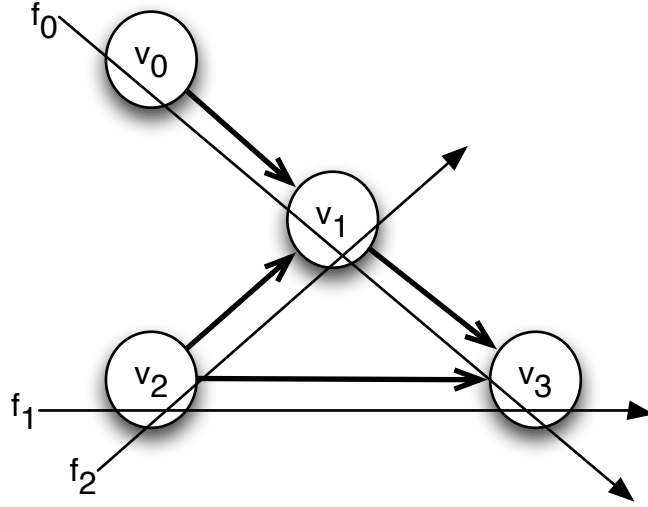


FeedForward_1SC_3Flows_1AC_3Paths



- $\beta_{v_0} = \beta_{v_1} = \beta_{v_2} = \beta_{v_3} = \beta_{R_{v_i}, T_{v_i}} = \beta_{20,20}, i \in \{0, 1, 3\}$
- $\alpha^{f_0} = \alpha^{f_1} = \alpha^{f_2} = \gamma_{r^{f_j}, b^{f_j}} = \gamma_{5,25}, j \in \{0, 1, 2\}$

The above topology can be transformed into an equivalent one by removing the unused edges:



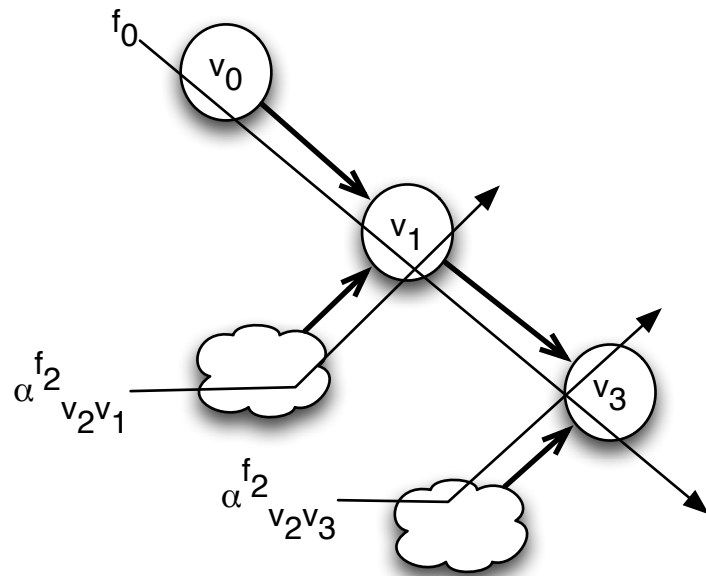
computeOutputBound(v_2, f_i) = $(\alpha_{v_2}^{f_i})^*, i \in \{1, 2\}$		FIFO_MUX	ARB_MUX
$\alpha_{v_2}^{xf_i} = \alpha_{v_2}^{xf_1} = \alpha_{v_2}^{xf_2} = \alpha^{f_1} = \alpha^{f_2}$		$= \gamma_{5,25}$	
$\beta_{v_2}^{l.o.f_i} = [\beta_{v_2} - \alpha_{v_2}^{xf_i}]^+ = \beta_{R_{v_2}^{l.o.f_i}, T_{v_2}^{l.o.f_i}}$	$R_{v_2}^{l.o.f_i}$	$[R_{v_2} - r_{v_2}^{xf_i}]^+ = 15$	
	$T_{v_2}^{l.o.f_i}$	$\beta_{v_2} = b_{v_2}^{xf_i}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{v_2} = \alpha_{v_2}^{xf_i}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15, 21\frac{1}{4}}$	$= \beta_{15, 28\frac{1}{3}}$
	$=$	$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$
$(\alpha_{v_2}^{f_i})^* = \alpha^{f_i} \oslash \beta_{v_2}^{l.o.f_i} = \gamma_{(r_{v_2}^{f_i})^*, (b_{v_2}^{f_i})^*}$	$(r_{v_2}^{f_i})^*$	$= 5$	
	$(b_{v_2}^{f_i})^*$	$\alpha^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 21\frac{1}{4} + 25 = 131\frac{1}{4}$	$\alpha^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 28\frac{1}{3} + 25 = 166\frac{2}{3}$
	$=$	$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$

$\text{computeOutputBound}(v_0, f_0) = (\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{x f_0}$		$= \gamma_{0,0}$	
$\beta_{v_0}^{\text{l.o.}, f_0} = [\beta_{v_0} - \alpha_{v_0}^{x f_0}]^+ = \beta_{v_0}$		$= \beta_{20,20}$	
$(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0} = \alpha^{f_0} \odot \beta_{v_0} = \gamma_{r_{v_0 v_1}^{f_0}, b_{v_0 v_1}^{f_0}}$	$r_{v_0 v_1}^{f_0}$	$= 5$	
	$b_{v_0 v_1}^{f_0}$	$\alpha^{f_0}(T_{v_0}) = 5 \cdot 20 + 25 = 125$	
	$=$	$= \gamma_{5,125}$	

$\text{computeOutputBound}(v_1, f_0) = (\alpha_{v_1}^{f_0})^* = \alpha_{v_1 v_3}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_1}^{x f_0} = \alpha_{v_2 v_1}^{f_2} = (\alpha_{v_2}^{f_2})^*$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\beta_{v_1}^{\text{l.o.}, f_0} = [\beta_{v_1} - \alpha_{v_1}^{x f_0}]^+ = \beta_{R_{v_1}^{\text{l.o.}, f_0}, T_{v_1}^{\text{l.o.}, f_0}}$	$R_{v_1}^{\text{l.o.}, f_0}$	$[R_{v_1} - r_{v_1}^{x f_0}]^+ = 15$	
	$T_{v_1}^{\text{l.o.}, f_0}$	$\beta_{v_1} = b_{v_1}^{x f_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{v_1} = \alpha_{v_1}^{x f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{8}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{8}}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{8}}$
$(\alpha_{v_1}^{f_0})^* = (\alpha_{v_0}^{f_0})^* \odot \beta_{v_1}^{\text{l.o.}, f_0} = \gamma_{r_{v_1 v_3}^{f_0}, b_{v_1 v_3}^{f_0}}$	$r_{v_1 v_3}^{f_0}$	$= 5$	
	$b_{v_1 v_3}^{f_0}$	$(\alpha_{v_0}^{f_0})^*(T_{v_1}^{\text{l.o.}, f_0}) = 5 \cdot 26\frac{9}{16} + 125 = 257\frac{13}{16}$	$(\alpha_{v_0 v_1}^{f_0})^*(T_{v_1}^{\text{l.o.}, f_0}) = 5 \cdot 37\frac{7}{8} + 125 = 313\frac{8}{9}$
	$=$	$= \gamma_{5,257\frac{13}{16}}$	$= \gamma_{5,313\frac{8}{9}}$

Flow f_0 (comparable to Tandem_1SC_2Flows_1SC_2Paths)

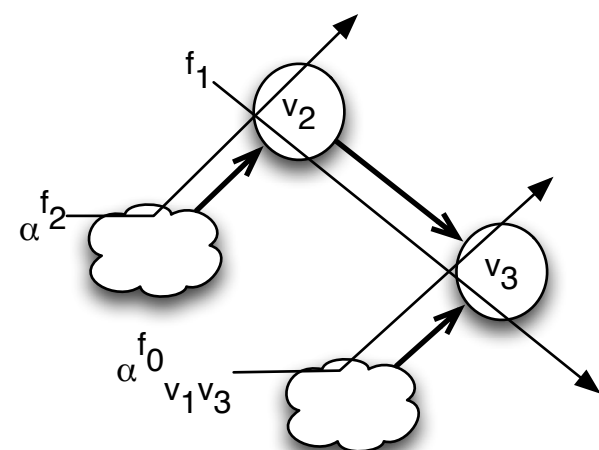
Output bound abstraction of f_0 's cross flows leads to a tree topology comparable to Tandem_1SC_2Flows_1SC_2Paths:



TFA		FIFO_MUX	ARB_MUX
v_0	$\alpha_{v_0} = \alpha_{v_0}^{f_0}$	$= \gamma_{5,25}$	
	$D_{v_0}^{f_0}$	$\beta_{v_0} = b_{v_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{v_0} = b_{v_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{v_0}^{f_0}$	$\alpha_{v_0}(T_{v_0}) = 20 \cdot 5 + 25$ $= 125$	
v_1	$\alpha_{v_1} = (\alpha_{v_0}^{f_0})^* + (\alpha_{v_2}^{f_2})^*$	$= \gamma_{5,125} + \gamma_{5,131\frac{1}{4}} = \gamma_{10,256\frac{1}{4}}$	$= \gamma_{5,125} + \gamma_{5,166\frac{2}{3}} = \gamma_{10,291\frac{2}{3}}$
	$D_{v_1}^{f_0}$	$\beta_{v_1} = b_{v_1}$ $20 \cdot [t - 20]^+ = 256\frac{1}{4}$ $t = 32\frac{13}{16}$	$\beta_{v_1} = \alpha_{v_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 291\frac{2}{3}$ $t = 69\frac{1}{6}$
	$B_{v_1}^{f_0}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 256\frac{1}{4}$ $= 456\frac{1}{4}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 291\frac{2}{3}$ $= 491\frac{2}{3}$
v_3	$\alpha_{v_3} = (\alpha_{v_1}^{f_0})^* + (\alpha_{v_2}^{f_1})^*$	$= \gamma_{5,257\frac{13}{16}} + \gamma_{5,131\frac{1}{4}} = \gamma_{10,389\frac{1}{16}}$	$= \gamma_{5,313\frac{8}{9}} + \gamma_{5,166\frac{2}{3}} = \gamma_{10,480\frac{5}{9}}$
	$D_{v_3}^{f_0}$	$\beta_{v_3} = b_{v_3}$ $20 \cdot [t - 20]^+ = 389\frac{1}{16}$ $t = 39\frac{29}{64}$	$\beta_{v_3} = \alpha_{v_3}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 480\frac{5}{9}$ $t = 88\frac{5}{90}$
	$B_{v_3}^{f_0}$	$\alpha_{v_3}(T_{v_3}) = 10 \cdot 20 + 389\frac{1}{16}$ $= 589\frac{1}{16}$	$\alpha_{v_3}(T_{v_3}) = 10 \cdot 20 + 480\frac{5}{9}$ $= 680\frac{5}{9}$
D^{f_0}		$\sum_{i=\{0,1,3\}} D_{v_i}^{f_0} = 93\frac{33}{64}$	$\sum_{i=\{0,1,3\}} D_{v_i}^{f_0} = 178\frac{17}{36}$
B^{f_0}		$\max_{i=\{0,1,3\}} b_{v_i}^{f_0} = 589\frac{1}{16}$	$\max_{i=\{0,1,3\}} b_{v_i}^{f_0} = 680\frac{5}{9}$

SFA, PMOO		FIFO_MUX (SFA only)	ARB_MUX
v_0	$\alpha_{v_0}^{xf_0} = \gamma_{r_{v_0}^{xf_0}, b_{v_0}^{xf_0}}$	$= \gamma_{0,0}$	
	$\beta_{v_0}^{l.o.f_0} = [\beta_{v_0} - \alpha_{v_0}^{xf_0}]^+$	$= \beta_{20,20}$	
v_1	$\alpha_{v_1}^{xf_0} = (\alpha_{v_2}^{f_1})^*$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
	$\beta_{v_1}^{l.o.f_0} = [\beta_{v_1} - \alpha_{v_1}^{xf_0}]^+$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{9}}$
v_3	$\alpha_{v_3}^{xf_0} = (\alpha_{v_2}^{f_2})^*$		$= \gamma_{5,131\frac{1}{4}}$
	$\beta_{v_3}^{l.o.f_0} = [\beta_{v_3} - \alpha_{v_3}^{xf_0}]^+ = \beta_{R_{v_3}^{l.o.f_0}, T_{v_3}^{l.o.f_0}}$		$= \gamma_{5,166\frac{2}{3}}$
	$R_{v_3}^{l.o.f_0}$	$[R_{v_3} - r_{v_3}^{xf_0}]^+ = 15$	
	$T_{v_3}^{l.o.f_0}$	$\beta_{v_3} = b_{v_3}^{xf_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{v_3} = \alpha_{v_3}^{xf_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{9}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{9}}$
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$		$\bigotimes_{i=\{0,1,3\}} \beta_{v_i}^{l.o.f_0} = \beta_{15,73\frac{1}{8}}$	$\bigotimes_{i=\{0,1,3\}} \beta_{v_i}^{l.o.f_0} = \beta_{15,95\frac{5}{9}}$
D^{f_0}		$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 73\frac{1}{8}]^+ = 25$ $t = 74\frac{19}{24}$	$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $15 \cdot [t - 95\frac{5}{9}]^+ = 25$ $t = 97\frac{2}{9}$
B^{f_0}		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 73\frac{1}{8} + 25$ $= 390\frac{5}{8}$	$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 95\frac{5}{9} + 25$ $= 502\frac{7}{9}$

Flow f_1



TFA		FIFO_MUX	ARB_MUX
v_2	$\alpha_{v_2} = \alpha^{f_1} + \alpha^{f_2}$	$= \gamma_{10,50}$	
	$D_{v_2}^{f_1}$	$\beta_{v_2} = b_{v_2}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{v_2} = \alpha_{v_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{v_2}^{f_1}$	$\alpha_{v_2}(T_{v_2}) = 20 \cdot 10 + 50$ $= 250$	
v_3	$\alpha_{v_3} = (\alpha_{v_1}^{f_0})^* + (\alpha_{v_2}^{f_1})^*$	$= \gamma_{5,257\frac{13}{16}} + \gamma_{5,131\frac{1}{4}} = \gamma_{10,389\frac{1}{16}}$	
	$D_{v_3}^{f_1}$	$\beta_{v_3} = b_{v_3}$ $20 \cdot [t - 20]^+ = 389\frac{1}{16}$ $t = 39\frac{29}{64}$	$\beta_{v_3} = \alpha_{v_3}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 480\frac{5}{9}$ $t = 88\frac{5}{90}$
	$B_{v_3}^{f_1}$	$\alpha_{v_3}(T_{v_3}) = 10 \cdot 20 + 389\frac{1}{16}$ $= 589\frac{1}{16}$	
D^{f_1}		$\sum_{i=2}^3 D_{v_i}^{f_1} = 61\frac{61}{64}$	$\sum_{i=2}^3 D_{v_i}^{f_1} = 185\frac{5}{9}$
B^{f_1}		$\max_{i=\{2,3\}} b_{v_i}^{f_1} = 589\frac{1}{16}$	$\max_{i=\{2,3\}} b_{v_i}^{f_1} = 680\frac{5}{9}$

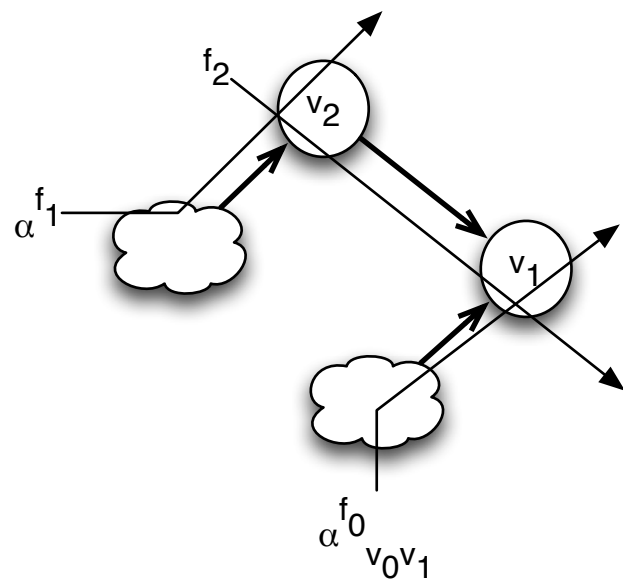
SFA Output Bounds: Consider the flow of interest f_0 to be of lowest priority, i.e., it does not decrease it's own cross-traffic's left-of-er service curve $\beta^{\text{SFA l.o. } xf_0}$ at any server.

SFA Output Bound(v_1, f_0) = $(\alpha_{v_1}^{f_0})^* = \alpha_{v_1 v_3}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_1}^{xf_0} = \alpha_{v_2 v_1}^{f_2} = (\alpha_{v_2}^{f_2})^* \text{ (SFA)} = \gamma_{5,25} \oslash \beta_{20,20}$		$= \gamma_{5,125}$	
$\beta_{v_1}^{\text{l.o.}f_0} = [\beta_{v_1} - \alpha_{v_1}^{xf_0}]^+ = \beta_{R_{v_1}^{\text{l.o.}f_0}, T_{v_1}^{\text{l.o.}f_0}}$	$R_{v_1}^{\text{l.o.}f_0}$	$[R_{v_1} - r_{v_1}^{xf_0}]^+ = 15$	
	$T_{v_1}^{\text{l.o.}f_0}$	$\beta_{v_1} = b_{v_1}^{xf_0}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$\beta_{v_1} = \alpha_{v_1}^{xf_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
	$=$	$= \beta_{15, 26\frac{1}{4}}$	$= \beta_{15, 35}$
	$r_{v_1 v_3}^{f_0}$	$= 5$	
$(\alpha_{v_1}^{f_0})^* = (\alpha_{v_0}^{f_0})^* \oslash \beta_{v_1}^{\text{l.o.}f_0} = \gamma_{r_{v_1 v_3}^{f_0}, b_{v_1 v_3}^{f_0}}$	$b_{v_1 v_3}^{f_0}$	$(\alpha_{v_0}^{f_0})^*(T_{v_1}^{\text{l.o.}f_0}) = 5 \cdot 26\frac{1}{4} + 125 = 256\frac{1}{4}$	$(\alpha_{v_0 v_1}^{f_0})^*(T_{v_1}^{\text{l.o.}f_0}) = 5 \cdot 35 + 125 = 300$
	$=$	$= \gamma_{5, 256\frac{1}{4}}$	$= \gamma_{5, 300}$

SFA			FIFO_MUX	ARB_MUX
v2	$\alpha_{v_2}^{xf_1} = \alpha_{v_2}^{f_2} = \alpha^{f_2}$		$= \gamma_{5,25}$	
	$\beta_{v_2}^{l.o.f_1} = [\beta_{v_2} - \alpha_{v_2}^{xf_1}]^+$		$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
v3	$\alpha_{v_3}^{xf_1} = \alpha_{v_3}^{f_0} = \alpha_{v_1v_3}^{f_0}$		$= \gamma_{5,256\frac{1}{4}}$	$= \gamma_{5,300}$
	$\beta_{v_3}^{l.o.f_1} = [\beta_{v_3} - \alpha_{v_3}^{xf_1}]^+ = \beta_{R_{v_3}^{l.o.f_1}, T_{v_3}^{l.o.f_1}}$	$R_{v_3}^{l.o.f_1}$	$[R_{v_3} - r_{v_3}^{xf_1}]^+ = 15$	
		$T_{v_3}^{l.o.f_1}$	$\beta_{v_3} = b_{v_3}^{xf_1}$	$\beta_{v_3} = \alpha_{v_3}^{xf_1}$
			$20 \cdot [t - 20]^+ = 256\frac{1}{4}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 300$
		$t = 32\frac{13}{16}$	$t = 46\frac{2}{3}$	
	$=$	$= \beta_{15,32\frac{13}{16}}$	$= \beta_{15,46\frac{2}{3}}$	
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$			$\bigotimes_{i=2}^3 \beta_{v_i}^{l.o.f_1} = \beta_{15,54\frac{1}{16}}$	$\bigotimes_{i=2}^3 \beta_{v_i}^{l.o.f_1} = \beta_{15,75}$
D^{f_1}			$\beta_{e2e}^{l.o.f_1} = b^{f_1}$	$\beta_{e2e}^{l.o.f_1} = b^{f_1}$
			$15 \cdot [t - 54\frac{1}{16}]^+ = 25$	$15 \cdot [t - 75]^+ = 25$
			$t = 55\frac{35}{48}$	$t = 76\frac{2}{3}$
B^{f_1}			$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 54\frac{1}{16} + 25$	$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 75 + 25$
			$= 295\frac{5}{16}$	$= 400$

PMOO			ARB_MUX	
v_2	$\alpha_{v_2}^{xf_1} = \alpha_{v_2}^{f_2} = \alpha^{f_2}$		$= \gamma_{5,25}$	
	$\beta_{v_2}^{l.o.f_1} = [\beta_{v_2} - \alpha_{v_2}^{xf_1}]^+$		$= \beta_{15,28\frac{1}{3}}$	
v_3	$\alpha_{v_3}^{xf_1} = \alpha_{v_3}^{f_0} = \alpha_{v_1v_3}^{f_0}$		$= \gamma_{5,313\frac{8}{9}}$	
	$\beta_{v_3}^{l.o.f_1} = [\beta_{v_3} - \alpha_{v_3}^{xf_1}]^+ = \beta_{R_{v_3}^{l.o.f_1}, T_{v_3}^{l.o.f_1}}$	$R_{v_3}^{l.o.f_1}$	$[R_{v_3} - r_{v_3}^{xf_1}]^+ = 15$	
		$T_{v_3}^{l.o.f_1}$	$\beta_{v_3} = \alpha_{v_3}^{xf_1}$	
		$=$	$20 \cdot [t - 20]^+ = 5 \cdot t + 313\frac{8}{9}$	
			$t = 47\frac{16}{27}$	
			$= \beta_{15,47\frac{16}{27}}$	
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$			$\bigotimes_{i=2}^3 \beta_{v_i}^{l.o.f_1} = \beta_{15,75\frac{25}{27}}$	
D^{f_1}			$\beta_{e2e}^{l.o.f_1} = b^{f_1}$	
			$15 \cdot [t - 75\frac{25}{27}]^+ = 25$	
			$t = 77\frac{16}{27}$	
B^{f_1}			$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 75\frac{25}{27} + 25$	
			$= 404\frac{17}{27}$	

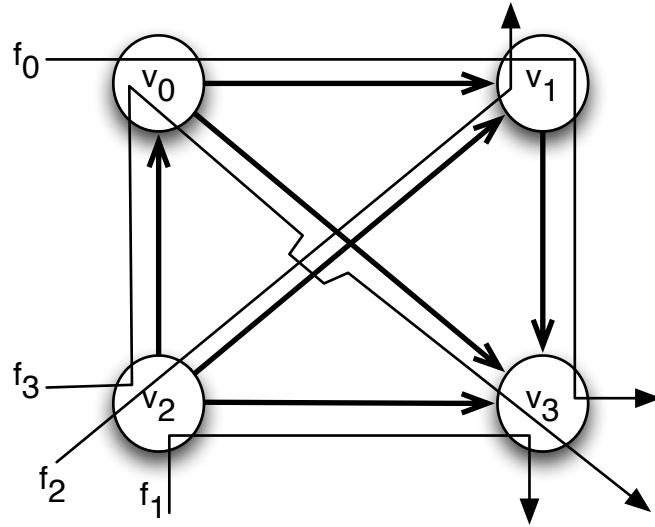
Flow f_2



TFA		FIFO_MUX	ARB_MUX
v_2	$\alpha_{v_2} = \alpha^{f_1} + \alpha^{f_2}$	$= \gamma_{10,50}$	
	$D_{v_2}^{f_2}$	$\beta_{v_2} = b_{v_2}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{v_2} = \alpha_{v_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{v_2}^{f_2}$	$\alpha_{v_2}(T_{v_2}) = 20 \cdot 10 + 50$ $= 250$	
v_1	$\alpha_{v_3} = (\alpha_{v_0}^{f_0})^* + (\alpha_{v_2}^{f_1})^*$	$= \gamma_{5,125} + \gamma_{5,131\frac{1}{4}} = \gamma_{10,256\frac{1}{4}}$	$= \gamma_{5,125} + \gamma_{5,166\frac{2}{3}} = \gamma_{10,291\frac{2}{3}}$
	$D_{v_1}^{f_2}$	$\beta_{v_1} = b_{v_1}$ $20 \cdot [t - 20]^+ = 256\frac{1}{4}$ $t = 32\frac{13}{16}$	$\beta_{v_1} = \alpha_{v_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 291\frac{2}{3}$ $t = 69\frac{1}{6}$
	$B_{v_1}^{f_2}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 256\frac{1}{4}$ $= 456\frac{1}{4}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 291\frac{2}{3}$ $= 491\frac{2}{3}$
D^{f_2}		$\sum_{i=1}^2 D_{v_i}^{f_2} = 55\frac{5}{16}$	$\sum_{i=1}^2 D_{v_i}^{f_2} = 114\frac{1}{6}$
B^{f_2}		$\max_{i=\{1,2\}} b_{v_i}^{f_2} = 456\frac{1}{4}$	$\max_{i=\{1,2\}} b_{v_i}^{f_2} = 491\frac{2}{3}$

SFA, PMOO		FIFO_MUX (SFA only)	ARB_MUX
v_2	$\alpha_{v_2}^{xf_2} = \alpha_{v_2}^{f_1} = \alpha^{f_1}$	$= \gamma_{5,25}$	
	$\beta_{v_2}^{l.o.f_2} = [\beta_{v_2} - \alpha_{v_2}^{xf_1}]^+$	$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$
v_1	$\alpha_{v_1}^{xf_1} = (\alpha_{v_0}^{f_0})^*$		$= \gamma_{5,125}$
	$\beta_{v_1}^{l.o.f_2} = [\beta_{v_1} - \alpha_{v_1}^{xf_2}]^+ = \beta_{R_{v_1}^{l.o.f_2}, T_{v_1}^{l.o.f_2}}$	$R_{v_1}^{l.o.f_2}$	$[R_{v_1} - r_{v_1}^{xf_2}]^+ = 15$
		$\beta_{v_1} = b_{v_1}$	$\beta_{v_1} = \alpha_{v_1}^{xf_2}$
		$20 \cdot [t - 20]^+ = 125$	$20 \cdot [t - 20]^+ = 5 \cdot t + 125$
		$t = 26\frac{1}{4}$	$t = 35$
	$=$		$= \beta_{15,26\frac{1}{4}}$
$\beta_{e2e}^{l.o.f_2} = \beta_{R_{e2e}^{l.o.f_2}, T_{e2e}^{l.o.f_2}}$		$\bigotimes_{i=1}^2 \beta_{v_i}^{l.o.f_2} = \beta_{15,47\frac{1}{2}}$	$\bigotimes_{i=1}^2 \beta_{v_i}^{l.o.f_2} = \beta_{15,63\frac{1}{3}}$
D^{f_2}		$\beta_{e2e}^{l.o.f_2} = b^{f_2}$	$\beta_{e2e}^{l.o.f_2} = b^{f_2}$
		$15 \cdot [t - 47\frac{1}{2}]^+ = 25$	$15 \cdot [t - 63\frac{1}{3}]^+ = 25$
		$t = 49\frac{1}{6}$	$t = 65$
B^{f_2}		$\alpha^{f_2}(T_{e2e}^{l.o.f_2}) = 5 \cdot 47\frac{1}{2} + 25$	$\alpha^{f_1}(T_{e2e}^{l.o.f_2}) = 5 \cdot 63\frac{1}{3} + 25$
		$= 262\frac{1}{2}$	$= 341\frac{2}{3}$

FeedForward_1SC_4Flows_1AC_4Paths



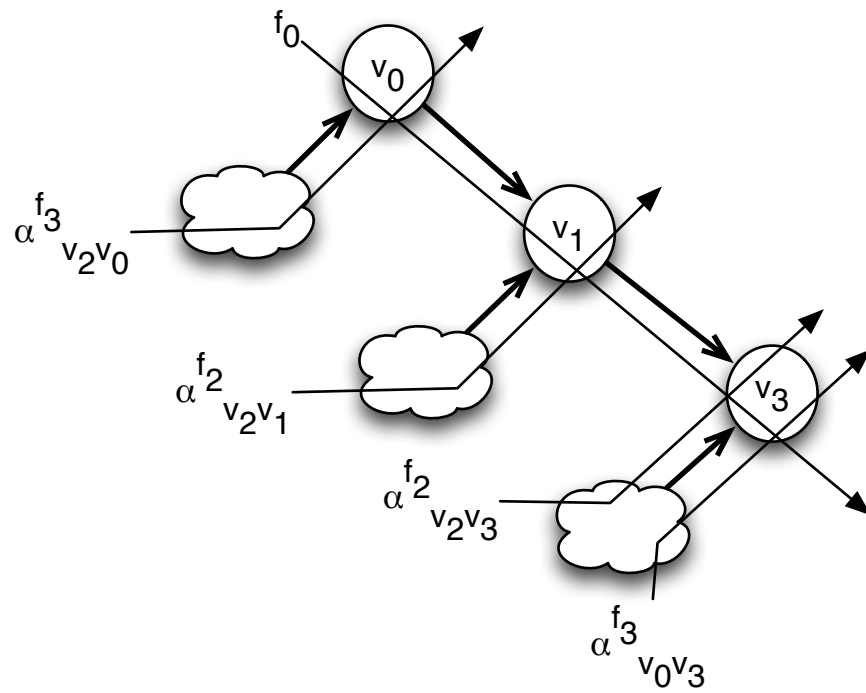
- $\beta_{v_0} = \beta_{v_1} = \beta_{v_2} = \beta_{v_3} = \beta_{R_{v_i}, T_{v_i}} = \beta_{20,20}, i \in \{0, 1, 3\}$
- $\alpha^{f_i} = \gamma_{r^{f_j}, b^{f_j}} = \gamma_{5,25}, j \in \{0, 3\}$

TFA Output Bounds:

computeOutputBound(v_2, f_i) = $(\alpha_{v_2}^{f_i})^*, i \in \{1, 2, 3\}$		FIFO_MUX	ARB_MUX
$\alpha_{v_2}^{x f_i} = \alpha_{v_2}^{x f_1} = \alpha_{v_2}^{x f_2} = \alpha_{v_2}^{x f_3}$		$= \gamma_{10,50}$	
$\beta_{v_2}^{l.o.f_i} = [\beta_{v_2} - \alpha_{v_2}^{x f_i}]^+ = \beta_{R_{v_2}^{l.o.f_i}, T_{v_2}^{l.o.f_i}}$	$R_{v_2}^{l.o.f_i}$	$[R_{v_2} - r_{v_2}^{x f_i}]^+ = 15$	
	$T_{v_2}^{l.o.f_i}$	$\beta_{v_2} = b_{v_2}^{x f_i}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{v_2} = \alpha_{v_2}^{x f_i}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$=$	$= \beta_{10,22\frac{1}{2}}$	$= \beta_{10,45}$
	$(r_{v_2}^{f_i})^*$	$= 5$	
$(\alpha_{v_2}^{f_i})^* = \alpha^{f_i} \oslash \beta_{v_2}^{l.o.f_i} = \gamma_{(r_{v_2}^{f_i})^*, (b_{v_2}^{f_i})^*}$	$(b_{v_2}^{f_i})^*$	$\alpha^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 22\frac{1}{2} + 25 = 137\frac{1}{2}$	$\alpha^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 45 + 25 = 250$
	$=$	$= \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,250}$
computeOutputBound(v_0, f_0) = $(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{x f_0} = \alpha_{v_2 v_0}^{f_3} = (\alpha_{v_2}^{f_3})^*$		$= \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,250}$
$\beta_{v_0}^{l.o.f_0} = [\beta_{v_0} - \alpha_{v_0}^{x f_0}]^+ = \beta_{R_{v_0}^{l.o.f_0}, T_{v_0}^{l.o.f_0}}$	$R_{v_0}^{l.o.f_0}$	$[R_{v_0} - r_{v_0}^{x f_0}]^+ = 15$	
	$T_{v_0}^{l.o.f_0}$	$\beta_{v_0} = b_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 137\frac{1}{2}$ $t = 26\frac{7}{8}$	$\beta_{v_0} = \alpha_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 250$ $t = 43\frac{1}{3}$
	$=$	$= \beta_{15,26\frac{7}{8}}$	$= \beta_{15,43\frac{1}{3}}$
	$r_{v_0 v_1}^{f_0}$	$= 5$	
$(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0} = \alpha^{f_0} \oslash \beta_{v_0}^{l.o.f_0} = \gamma_{r_{v_0 v_1}^{f_0}, b_{v_0 v_1}^{f_0}}$	$b_{v_0 v_1}^{f_0}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 26\frac{7}{8} + 25 = 159\frac{3}{8}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 43\frac{1}{3} + 25 = 241\frac{2}{3}$
	$=$	$= \gamma_{5,159\frac{3}{8}}$	$= \gamma_{5,241\frac{2}{3}}$

computeOutputBound(v_0, f_3) = $(\alpha_{v_0}^{f_3})^* = \alpha_{v_0 v_3}^{f_3}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{x f_3} = \alpha^{f_0}$		$= \gamma_{5,25}$	
$\beta_{v_0}^{l.o.f_3} = [\beta_{v_0} - \alpha_{v_0}^{x f_3}]^+ = [\beta_{v_0} - \alpha^{f_0}]^+ = \beta_{R_{v_0}^{l.o.f_3}, T_{v_0}^{l.o.f_3}}$	$R_{v_0}^{l.o.f_3}$	$[R_{v_0} - r_{v_0}^{x f_3}]^+ = 15$	
	$T_{v_0}^{l.o.f_3}$	$\beta_{v_0} = b_{v_0}^{f_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{v_0} = \alpha^{f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15, 21\frac{1}{4}}$	$= \beta_{15, 28\frac{1}{3}}$
	$r_{v_0 v_3}^{f_3}$	$= 5$	
$(\alpha_{v_0}^{f_3})^* = (\alpha_{v_2}^{f_3})^* \odot \beta_{v_0}^{l.o.f_3} = \gamma_{r_{v_0 v_3}^{f_3}, b_{v_0 v_3}^{f_3}}$	$b_{v_0 v_3}^{f_3}$	$(\alpha_{v_2}^{f_3})^*(T_{v_0}^{l.o.f_3}) = 5 \cdot 21\frac{1}{4} + 137\frac{1}{2} = 243\frac{3}{4}$	$(\alpha_{v_2}^{f_3})^*(T_{v_0}^{l.o.f_3}) = 5 \cdot 28\frac{1}{3} + 250 = 391\frac{2}{3}$
	$=$	$= \gamma_{5, 243\frac{3}{4}}$	$= \gamma_{5, 391\frac{2}{3}}$
computeOutputBound(v_1, f_0) = $(\alpha_{v_1}^{f_0})^* = \alpha_{v_1 v_3}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_1}^{x f_0} = \alpha_{v_2 v_1}^{f_2} = (\alpha_{v_2}^{f_2})^*$		$= \gamma_{5, 137\frac{1}{2}}$	$= \gamma_{5, 250}$
$\beta_{v_1}^{l.o.f_0} = [\beta_{v_1} - \alpha_{v_1}^{x f_0}]^+ = \beta_{R_{v_1}^{l.o.f_0}, T_{v_1}^{l.o.f_0}}$	$R_{v_1}^{l.o.f_0}$	$[R_{v_1} - r_{v_1}^{x f_0}]^+ = 15$	
	$T_{v_1}^{l.o.f_0}$	$\beta_{v_1} = b_{v_1}^{x f_0}$ $20 \cdot [t - 20]^+ = 137\frac{1}{2}$ $t = 26\frac{7}{8}$	$\beta_{v_1} = \alpha_{v_1}^{x f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 250$ $t = 43\frac{1}{3}$
	$=$	$= \beta_{15, 26\frac{7}{8}}$	$= \beta_{15, 43\frac{1}{3}}$
	$r_{v_1 v_3}^{f_0}$	$= 5$	
$(\alpha_{v_1}^{f_0})^* = (\alpha_{v_0}^{f_0})^* \odot \beta_{v_1}^{l.o.f_0} = \gamma_{r_{v_1 v_3}^{f_0}, b_{v_1 v_3}^{f_0}}$	$b_{v_1 v_3}^{f_0}$	$(\alpha_{v_0}^{f_0})^*(T_{v_1}^{l.o.f_0}) = 5 \cdot 26\frac{7}{8} + 159\frac{3}{8} = 293\frac{3}{4}$	$(\alpha_{v_0}^{f_0})^*(T_{v_1}^{l.o.f_0}) = 5 \cdot 43\frac{1}{3} + 241\frac{2}{3} =$
	$=$	$= \gamma_{5, 293\frac{3}{4}}$	$= \gamma_{5, 458\frac{1}{3}}$

Flow f_0



Correction at v_3 : There need to be $\alpha^{f_1}_{v_2 v_3}$ instead of $\alpha^{f_2}_{v_2 v_3}$

TFA		FIFO_MUX	ARB_MUX
v_0	$\alpha_{v_0} = \alpha^{f_0} + \alpha_{v_2 v_0}^{f_3} = \alpha^{f_0} + (\alpha_{v_2}^{f_3})^*$	$= \gamma_{5,25} + \gamma_{5,137\frac{1}{2}} = \gamma_{10,162\frac{1}{2}}$	$= \gamma_{5,25} + \gamma_{5,250} = \gamma_{10,275}$
	$D_{v_0}^{f_0}$	$\beta_{v_0} = b_{v_0}$ $20 \cdot [t - 20]^+ = 162\frac{1}{2}$ $t = 28\frac{1}{8}$	$\beta_{v_0} = \alpha_{v_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 275$ $t = 67\frac{1}{2}$
	$B_{v_0}^{f_0}$	$\alpha_{v_0}(T_{v_0}) = 10 \cdot 20 + 162\frac{1}{2}$ $= 262\frac{1}{2}$	$\alpha_{v_0}(T_{v_0}) = 10 \cdot 20 + 275$ $= 475$
v_1	$\alpha_{v_1} = \alpha_{v_0 v_1}^{f_0} + \alpha_{v_2 v_1}^{f_2} = (\alpha_{v_0}^{f_0})^* + (\alpha_{v_2}^{f_1})^*$	$= \gamma_{5,159\frac{3}{8}} + \gamma_{5,137\frac{1}{2}} = \gamma_{10,296\frac{7}{8}}$	$= \gamma_{5,241\frac{2}{3}} + \gamma_{5,250} = \gamma_{10,491\frac{2}{3}}$
	$D_{v_1}^{f_0}$	$\beta_{v_1} = b_{v_1}$ $20 \cdot [t - 20]^+ = 296\frac{7}{8}$ $t = 34\frac{27}{32}$	$\beta_{v_1} = \alpha_{v_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 491\frac{2}{3}$ $t = 89\frac{1}{6}$
	$B_{v_1}^{f_0}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 296\frac{7}{8}$ $= 496\frac{7}{8}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 491\frac{2}{3}$ $= 691\frac{2}{3}$
v_3	$\alpha_{v_3} = (\alpha_{v_1}^{f_0})^* + (\alpha_{v_2}^{f_1})^* + (\alpha_{v_0}^{f_3})^*$	$= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}} = \gamma_{15,675}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,391\frac{2}{3}} = \gamma_{15,1100}$
	$D_{v_3}^{f_0}$	$\beta_{v_3} = b_{v_3}$ $20 \cdot [t - 20]^+ = 675$ $t = 53\frac{3}{4}$	$\beta_{v_3} = \alpha_{v_3}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 1100$ $t = 300$
	$B_{v_3}^{f_0}$	$\alpha_{v_3}(T_{v_3}) = 15 \cdot 20 + 675$ $= 975$	$\alpha_{v_3}(T_{v_3}) = 15 \cdot 20 + 1100$ $= 1400$
D^{f_0}		$D_{v_0}^{f_0} + D_{v_1}^{f_0} + D_{v_3}^{f_0} = 116\frac{23}{32}$	$D_{v_0}^{f_0} + D_{v_1}^{f_0} + D_{v_3}^{f_0} = 456\frac{2}{3}$
B^{f_0}		$\max\{B_{v_0}^{f_0}, B_{v_1}^{f_0}, B_{v_3}^{f_0}\} = 975$	$\max\{B_{v_0}^{f_0}, B_{v_1}^{f_0}, B_{v_3}^{f_0}\} = 1400$

SFA Output Bound differing from TFA one:

computeOutputBound(v_0, f_3) = $(\alpha_{v_0}^{f_3})^* = \alpha_{v_0 v_3}^{f_3}$			FIFO_MUX	ARB_MUX	
$\alpha_{v_0}^{x f_3}$			$= \gamma_{0,0}$		
$\beta_{v_0}^{l.o.f_3} = \beta_{v_0} = \beta_{R_{v_0}^{l.o.f_3}, T_{v_0}^{l.o.f_3}}$			$= \beta_{20,20}$		
$(\alpha_{v_0}^{f_3})^* = (\alpha_{v_2}^{f_3})^* \odot \beta_{v_0}^{l.o.f_3} = \gamma_{r_{v_0 v_3}^{f_3}, b_{v_0 v_3}^{f_3}}$			$= 5$		
			$r_{v_0 v_3}^{f_3}$	$(\alpha_{v_2}^{f_3})^*(T_{v_0}^{l.o.f_3}) = 5 \cdot 20 + 137\frac{1}{2} = 237\frac{1}{2}$	$(\alpha_{v_2}^{f_3})^*(T_{v_0}^{l.o.f_3}) = 5 \cdot 20 + 250 = 350$
			$b_{v_0 v_3}^{f_3}$		
$=$			$= \gamma_{5,237\frac{1}{2}}$	$= \gamma_{5,350}$	
SFA			FIFO_MUX	ARB_MUX	
v_0	$\alpha_{v_0}^{x f_0} = (\alpha_{v_2}^{f_3})^*$		$= \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,250}$	
	$\beta_{v_0}^{l.o.f_0} = [\beta_{v_0} - \alpha_{v_0}^{x f_3}]^+$		$= \beta_{15,26\frac{7}{8}}$	$= \beta_{15,43\frac{1}{3}}$	
v_1	$\alpha_{v_1}^{x f_0} = (\alpha_{v_2}^{f_1})^*$		$= \gamma_{5,137\frac{1}{2}}$	$= \gamma_{5,250}$	
	$\beta_{v_1}^{l.o.f_0} = [\beta_{v_1} - \alpha_{v_1}^{x f_0}]^+$		$= \beta_{15,26\frac{7}{8}}$	$= \beta_{15,43\frac{1}{3}}$	
v_3	$\alpha_{v_3}^{x f_0} = (\alpha_{v_2}^{f_2})^* + (\alpha_{v_0}^{f_3})^*$		$= \gamma_{5,137\frac{1}{2}} + \gamma_{5,237\frac{1}{2}} = \gamma_{10,375}$	$= \gamma_{5,250} + \gamma_{5,350} = \gamma_{10,600}$	
	$\beta_{v_3}^{l.o.f_0} = [\beta_{v_3} - \alpha_{v_3}^{x f_0}]^+ = \beta_{R_{v_3}^{l.o.f_0}, T_{v_3}^{l.o.f_0}}$	$R_{v_3}^{l.o.f_0}$	$[R_{v_3} - r_{v_3}^{x f_0}]^+ = 10$		
		$T_{v_3}^{l.o.f_0}$	$\beta_{v_3} = b_{v_3}^{x f_0}$	$\beta_{v_3} = \alpha_{v_3}^{x f_0}$	
			$20 \cdot [t - 20]^+ = 375$	$20 \cdot [t - 20]^+ = 10 \cdot t + 600$	
			$t = 38\frac{3}{4}$	$t = 100$	
$=$		$= \beta_{10,38\frac{3}{4}}$	$= \beta_{10,100}$		
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$			$\bigotimes_{i=\{0,1,3\}} \beta_{v_i}^{l.o.f_0} = \beta_{10,92\frac{1}{2}}$	$\bigotimes_{i=\{0,1,3\}} \beta_{v_i}^{l.o.f_0} = \beta_{10,186\frac{2}{3}}$	
D^{f_0}			$\beta_{e2e}^{l.o.f_0} = b^{f_0}$	$\beta_{e2e}^{l.o.f_0} = b^{f_0}$	
			$10 \cdot [t - 92\frac{1}{2}]^+ = 25$	$10 \cdot [t - 186\frac{2}{3}]^+ = 25$	
			$t = 95$	$t = 189\frac{1}{6}$	
B^{f_0}			$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 92\frac{1}{2} + 25$	$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 186\frac{2}{3} + 25$	
			$= 487\frac{1}{2}$	$= 958\frac{1}{3}$	

PMOO (See [1] for details)

Cross traffic at node v_0 :

$$\alpha_{v_0}^{xf_0} = \alpha_{v_2v_0}^{f_3} = \gamma_{5,250}$$

(see $\text{computeOutputBound}(v_2, f_i) = (\alpha_{v_2}^{f_i})^*$, $i \in \{1, 2, 3\}$ above because f_3 only crosses a single server)

Cross traffic at node v_1 :

$$\alpha_{v_1}^{xf_0} = \alpha_{v_2v_1}^{f_2} = \gamma_{5,250}$$

(see $\text{computeOutputBound}(v_2, f_i) = (\alpha_{v_2}^{f_i})^*$, $i \in \{1, 2, 3\}$ above because f_2 only crosses a single server)

Cross traffic at node v_3 :

$$\alpha_{v_3}^{xf_0} = \alpha_{v_2v_3}^{f_1} + \alpha_{v_0v_3}^{f_3} = \gamma_{10,662\frac{1}{2}} \text{ with}$$

$$\alpha_{v_2v_3}^{f_1} = \gamma_{5,250}$$

(see $\text{computeOutputBound}(v_2, f_i) = (\alpha_{v_2}^{f_i})^*$, $i \in \{1, 2, 3\}$ above because f_1 only crosses a single server),

$$\alpha_{v_0v_3}^{f_3} = \alpha_{v_0}^{f_3} \oslash \beta_{v_2v_0}^{\text{PMOO l.o.}f_3}$$

$$\begin{aligned} R_{v_2v_0}^{\text{PMOO l.o.}f_3} &= (R_{v_2} - r_{v_2}^{xf_3}) \wedge (R_{v_0} - r_{v_0}^{xf_3}) \\ &= (20 - 10) \wedge (20 - 5) \\ &= 10 \end{aligned}$$

$$\begin{aligned} T_{v_2v_0}^{\text{PMOO l.o.}f_3} &= T_{v_2} + T_{v_0} + \frac{b_{v_2}^{xf_3} + b_{v_0}^{xf_3} + r_{v_2}^{xf_3} \cdot T_{v_2} + r_{v_0}^{xf_3} \cdot T_{v_0}}{(R_{v_2} - r_{v_2}^{xf_3}) \wedge (R_{v_0} - r_{v_0}^{xf_3})} \\ &= 20 + 20 + \frac{50 + 25 + 10 \cdot 20 + 5 \cdot 20}{10} \\ &= 77\frac{1}{2} \end{aligned}$$

$$\alpha_{v_0v_3}^{f_3} = \gamma_{5,25} \oslash \beta_{10,77\frac{1}{2}} = \gamma_{5,412\frac{1}{2}}$$

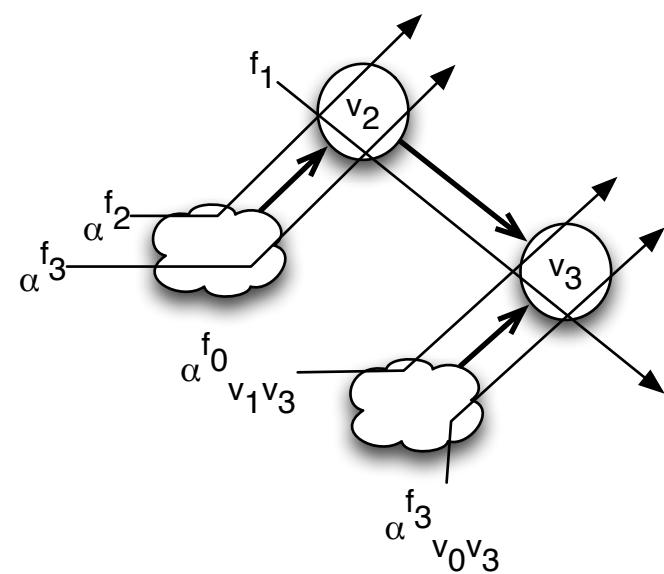
$$\beta^{\text{PMOO l.o.}f_0},$$

$$\begin{aligned} R^{\text{PMOO l.o.}f_0} &= (R_{v_0} - r_{v_0}^{xf_0}) \wedge (R_{v_1} - r_{v_1}^{xf_0}) \wedge (R_{v_3} - r_{v_3}^{xf_0}) \\ &= (20 - 5) \wedge (20 - 5) \wedge (20 - 10) \\ &= 10 \end{aligned}$$

$$\begin{aligned} T^{\text{PMOO l.o.}f_0} &= T_{v_0} + T_{v_1} + T_{v_3} + \frac{b_{v_0}^{xf_0} + b_{v_1}^{xf_0} + b_{v_3}^{xf_0} + r_{v_0}^{xf_0} \cdot T_{v_0} + r_{v_1}^{xf_0} \cdot T_{v_1} + r_{v_3}^{xf_0} \cdot T_{v_3}}{(R_{v_0} - r_{v_0}^{xf_0}) \wedge (R_{v_1} - r_{v_1}^{xf_0}) \wedge (R_{v_3} - r_{v_3}^{xf_0})} \\ &= 20 + 20 + 20 + \frac{250 + 250 + 662\frac{1}{2} + 5 \cdot 20 + 5 \cdot 20 + 10 \cdot 20}{10} \\ &= 60 + \frac{1562\frac{1}{2}}{10} \\ &= 216\frac{1}{4} \end{aligned}$$

$$\begin{aligned} D &= h(\alpha^{f_0}, \beta^{\text{PMOO l.o.}f_0}) = \frac{25}{10} + 216\frac{1}{4} = 218\frac{3}{4} \\ B &= v(\alpha^{f_0}, \beta^{\text{PMOO l.o.}f_0}) = 5 \cdot 216\frac{1}{4} + 25 = 1106\frac{1}{4} \end{aligned}$$

Flow f_1



TFA		FIFO_MUX	ARB_MUX
v_2	$\alpha_{v_1} = \alpha^{f_1} + \alpha^{f_2} + \alpha^{f_3}$	$= 3 \cdot \gamma_{5,25} = \gamma_{15,75}$	
	$D_{v_2}^{f_1}$	$\beta_{v_2} = b_{v_2}$ $20 \cdot [t - 20]^+ = 75$ $t = 23\frac{3}{4}$	$\beta_{v_2} = \alpha_{v_2}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 75$ $t = 95$
	$B_{v_2}^{f_1}$	$\alpha_{v_2}(T_{v_2}) = 15 \cdot 20 + 75$ $= 375$	
v_3	$\alpha_{v_3} = (\alpha_{v_2}^{f_1})^* + (\alpha_{v_1}^{f_0})^* + (\alpha_{v_0}^{f_3})^*$	$= \gamma_{5,137\frac{1}{2}} + \gamma_{5,293\frac{3}{4}} + \gamma_{5,243\frac{3}{4}} = \gamma_{15,675}$	$= \gamma_{5,250} + \gamma_{5,458\frac{1}{3}} + \gamma_{5,391\frac{2}{3}} = \gamma_{15,1100}$
	$D_{v_3}^{f_1}$	$\beta_{v_3} = b_{v_3}$ $20 \cdot [t - 20]^+ = 675$ $t = 53\frac{3}{4}$	$\beta_{v_3} = \alpha_{v_3}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 1100$ $t = 300$
	$B_{v_3}^{f_1}$	$\alpha_{v_3}(T_{v_3}) = 15 \cdot 20 + 675$ $= 975$	$\alpha_{v_3}(T_{v_3}) = 15 \cdot 20 + 1100$ $= 1400$
D^{f_1}		$D_{v_2}^{f_1} + D_{v_3}^{f_1} = 77\frac{1}{2}$	$D_{v_2}^{f_1} + D_{v_3}^{f_1} = 395$
B^{f_1}		$\max\{B_{v_2}^{f_1}, B_{v_3}^{f_1}\} = 975$	$\max\{B_{v_2}^{f_1}, B_{v_3}^{f_1}\} = 1400$

SFA Output Bound differing from TFA one:

computeOutputBound(v_2, f_i) = $(\alpha_{v_2}^{f_i})^*$, $i \in \{2, 3\}$		FIFO_MUX	ARB_MUX
$\alpha_{v_2}^{x f_i} = \alpha_{v_2}^{x f_2} = \alpha_{v_2}^{x f_3}$		$= \gamma_{5,25}$	
$\beta_{v_2}^{l.o.f_i} = [\beta_{v_2} - \alpha_{v_2}^{x f_i}]^+ = \beta_{R_{v_2}^{l.o.f_i}, T_{v_2}^{l.o.f_i}}$	$R_{v_2}^{l.o.f_i}$	$[R_{v_2} - r_{v_2}^{x f_i}]^+ = 15$	
	$T_{v_2}^{l.o.f_i}$	$\beta_{v_2} = b_{v_2}^{x f_i}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{v_2} = \alpha_{v_2}^{x f_i}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{10, 21\frac{1}{4}}$	$= \beta_{10, 28\frac{1}{3}}$
	$(r_{v_2}^{f_i})^*$	$= 5$	
$(\alpha_{v_2}^{f_i})^* = \alpha^{f_i} \oslash \beta_{v_2}^{l.o.f_i} = \gamma_{(r_{v_2}^{f_i})^*, (b_{v_2}^{f_i})^*}$	$(b_{v_2}^{f_i})^*$	$\alpha^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 21\frac{1}{4} + 25 = 126\frac{1}{4}$	$\alpha^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 28\frac{1}{3} + 25 = 166\frac{2}{3}$
	$=$	$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$
computeOutputBound(v_0, f_0) = $(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{x f_0} = \alpha_{v_2 v_0}^{f_3} = (\alpha_{v_2}^{f_3})^*$		$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$
$\beta_{v_0}^{l.o.f_0} = [\beta_{v_0} - \alpha_{v_0}^{x f_0}]^+ = \beta_{R_{v_0}^{l.o.f_0}, T_{v_0}^{l.o.f_0}}$	$R_{v_0}^{l.o.f_0}$	$[R_{v_0} - r_{v_0}^{x f_0}]^+ = 15$	
	$T_{v_0}^{l.o.f_0}$	$\beta_{v_0} = b_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{v_0} = \alpha_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{9}$
	$=$	$= \beta_{15, 26\frac{9}{16}}$	$= \beta_{15, 37\frac{7}{9}}$
	$r_{v_0 v_1}^{f_0}$	$= 5$	
$(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0} = \alpha^{f_0} \oslash \beta_{v_0}^{l.o.f_0} = \gamma_{r_{v_0 v_1}^{f_0}, b_{v_0 v_1}^{f_0}}$	$b_{v_0 v_1}^{f_0}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 26\frac{9}{16} + 25 = 157\frac{13}{16}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 37\frac{7}{9} + 25 = 213\frac{8}{9}$
	$=$	$= \gamma_{5, 157\frac{13}{16}}$	$= \gamma_{5, 213\frac{8}{9}}$

computeOutputBound(v_0, f_3) = $(\alpha_{v_0}^{f_3})^* = \alpha_{v_0 v_3}^{f_3}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{x f_3} = \alpha^{f_0}$		$= \gamma_{5,25}$	
$\beta_{v_0}^{l.o.f_3} = [\beta_{v_0} - \alpha_{v_0}^{x f_3}]^+ = [\beta_{v_0} - \alpha^{f_0}]^+ = \beta_{R_{v_0}^{l.o.f_3}, T_{v_0}^{l.o.f_3}}$	$R_{v_0}^{l.o.f_3}$	$[R_{v_0} - r_{v_0}^{x f_3}]^+ = 15$	
	$T_{v_0}^{l.o.f_3}$	$\beta_{v_0} = b_{v_0}^{f_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{v_0} = \alpha^{f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15, 21\frac{1}{4}}$	$= \beta_{15, 28\frac{1}{3}}$
	$r_{v_0 v_3}^{f_3}$	$= 5$	
$(\alpha_{v_0}^{f_3})^* = (\alpha_{v_2}^{f_3})^* \odot \beta_{v_0}^{l.o.f_3} = \gamma_{r_{v_0 v_3}^{f_3}, b_{v_0 v_3}^{f_3}}$	$b_{v_0 v_3}^{f_3}$	$(\alpha_{v_2}^{f_3})^*(T_{v_0}^{l.o.f_3}) = 5 \cdot 21\frac{1}{4} + 131\frac{1}{4} = 232\frac{1}{2}$	$(\alpha_{v_2}^{f_3})^*(T_{v_0}^{l.o.f_3}) = 5 \cdot 28\frac{1}{3} + 166\frac{2}{3} = 308\frac{1}{3}$
	$=$	$= \gamma_{5, 237\frac{1}{2}}$	$= \gamma_{5, 308\frac{1}{3}}$
computeOutputBound(v_1, f_0) = $(\alpha_{v_1}^{f_0})^* = \alpha_{v_1 v_3}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_1}^{x f_0} = \alpha_{v_2 v_1}^{f_2} = (\alpha_{v_2}^{f_2})^*$		$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$
$\beta_{v_1}^{l.o.f_0} = [\beta_{v_1} - \alpha_{v_1}^{x f_0}]^+ = \beta_{R_{v_1}^{l.o.f_0}, T_{v_1}^{l.o.f_0}}$	$R_{v_1}^{l.o.f_0}$	$[R_{v_1} - r_{v_1}^{x f_0}]^+ = 15$	
	$T_{v_1}^{l.o.f_0}$	$\beta_{v_0} = b_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{v_0} = \alpha_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{9}$
	$=$	$= \beta_{15, 26\frac{9}{16}}$	$= \beta_{15, 37\frac{7}{9}}$
	$r_{v_1 v_3}^{f_0}$	$= 5$	
$(\alpha_{v_1}^{f_0})^* = (\alpha_{v_0}^{f_0})^* \odot \beta_{v_1}^{l.o.f_0} = \gamma_{r_{v_1 v_3}^{f_0}, b_{v_1 v_3}^{f_0}}$	$b_{v_1 v_3}^{f_0}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 26\frac{9}{16} + 157\frac{13}{16} = 290\frac{5}{8}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 37\frac{7}{9} + 213\frac{8}{9} = 402\frac{7}{9}$
	$=$	$= \gamma_{5, 290\frac{5}{8}}$	$= \gamma_{5, 402\frac{7}{9}}$

SFA			FIFO_MUX	ARB_MUX
v_2	$\alpha_{v_2}^{xf_1} = \alpha^{f_2} + \alpha^{f_3}$		$= 2 \cdot \gamma_{5,25} = \gamma_{10,50}$	
	$\beta_{v_2}^{l.o.f_1} = [\beta_{v_2} - \alpha_{v_2}^{xf_1}]^+$		$= \beta_{10,22\frac{1}{2}}$	$= \beta_{10,45}$
v_3	$\alpha_{v_3}^{xf_1} = (\alpha_{v_1}^{f_0})^* + (\alpha_{v_0}^{f_3})^*$		$= \gamma_{5,290\frac{5}{8}} + \gamma_{5,237\frac{1}{2}} = \gamma_{10,528\frac{1}{8}}$	$= \gamma_{5,402\frac{7}{9}} + \gamma_{5,308\frac{1}{3}} = \gamma_{10,711\frac{1}{9}}$
	$\beta_{v_3}^{l.o.f_1} = [\beta_{v_3} - \alpha_{v_3}^{xf_1}]^+ = \beta_{R_{v_3}^{l.o.f_1}, T_{v_3}^{l.o.f_1}}$	$R_{v_3}^{l.o.f_1}$	$[R_{v_3} - r_{v_3}^{xf_1}]^+ = 10$	
		$T_{v_3}^{l.o.f_1}$	$\beta_{v_3} = b_{v_3}^{xf_1}$	$\beta_{v_3} = \alpha_{v_3}^{xf_1}$
			$20 \cdot [t - 20]^+ = 528\frac{1}{8}$	$20 \cdot [t - 20]^+ = 10 \cdot t + 711\frac{1}{9}$
			$t = 46\frac{13}{32}$	$t = 111\frac{1}{9}$
	$=$	$= \beta_{10,46\frac{13}{32}}$	$= \beta_{10,111\frac{1}{9}}$	
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$			$\bigotimes_{i=2}^3 \beta_{v_i}^{l.o.f_1} = \beta_{10,68\frac{29}{32}}$	$\bigotimes_{i=2}^3 \beta_{v_i}^{l.o.f_1} = \beta_{10,156\frac{1}{9}}$
D^{f_1}			$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $10 \cdot [t - 68\frac{29}{32}]^+ = 25$ $t = 71\frac{13}{32}$	$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $10 \cdot [t - 156\frac{1}{9}]^+ = 25$ $t = 158\frac{11}{18}$
B^{f_1}			$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 68\frac{29}{32} + 25$ $= 369\frac{17}{32}$	$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 156\frac{1}{9} + 25$ $= 805\frac{5}{9}$

PMOO (See [1] for details)

Cross traffic at node v_2 :

$$\alpha_{v_2}^{xf_1} = \alpha^{f_2} + \alpha^{f_3} = \gamma_{10,50}$$

Cross traffic at node v_3 :

$$\alpha_{v_3}^{xf_1} = \alpha_{v_0v_3}^{f_3} + \alpha_{v_1v_3}^{f_0} = \gamma_{10,870\frac{5}{6}} \text{ with}$$

$$\alpha_{v_0v_3}^{f_3} = \alpha^{f_3} \oslash \beta_{v_2v_0}^{\text{PMOO l.o.}f_3}$$

$$\begin{aligned} R_{v_2v_0}^{\text{PMOO l.o.}f_3} &= (R_{v_2} - r_{v_2}^{xf_3}) \wedge (R_{v_0} - r_{v_0}^{xf_3}) \\ &= (20 - 10) \wedge (20 - 5) \\ &= 10 \end{aligned}$$

$$\begin{aligned} T_{v_2v_0}^{\text{PMOO l.o.}f_3} &= T_{v_2} + T_{v_0} + \frac{b_{v_2}^{xf_3} + b_{v_0}^{xf_3} + r_{v_2}^{xf_3} \cdot T_{v_2} + r_{v_0}^{xf_3} \cdot T_{v_0}}{(R_{v_2} - r_{v_2}^{xf_3}) \wedge (R_{v_0} - r_{v_0}^{xf_3})} \\ &= 20 + 20 + \frac{50 + 25 + 10 \cdot 20 + 5 \cdot 20}{10} \\ &= 77\frac{1}{2} \end{aligned}$$

$$\alpha_{v_0v_3}^{f_3} = \gamma_{5,25} \oslash \beta_{10,77\frac{1}{2}} = \gamma_{5,412\frac{1}{2}}$$

$\alpha_{v_1v_3}^{f_0} = \alpha^{f_0} \oslash \beta_{v_0v_1}^{\text{PMOO l.o.}f_0}$ (for cross traffic see $\text{computeOutputBound}(v_2, f_i) = (\alpha_{v_2}^{f_i})^*$, $i \in \{1, 2, 3\}$ above because they only cross a single server before)

$$\begin{aligned} R_{v_0v_1}^{\text{PMOO l.o.}f_0} &= (R_{v_0} - r_{v_0}^{xf_0}) \wedge (R_{v_1} - r_{v_1}^{xf_0}) \\ &= (20 - 5) \wedge (20 - 5) \\ &= 15 \end{aligned}$$

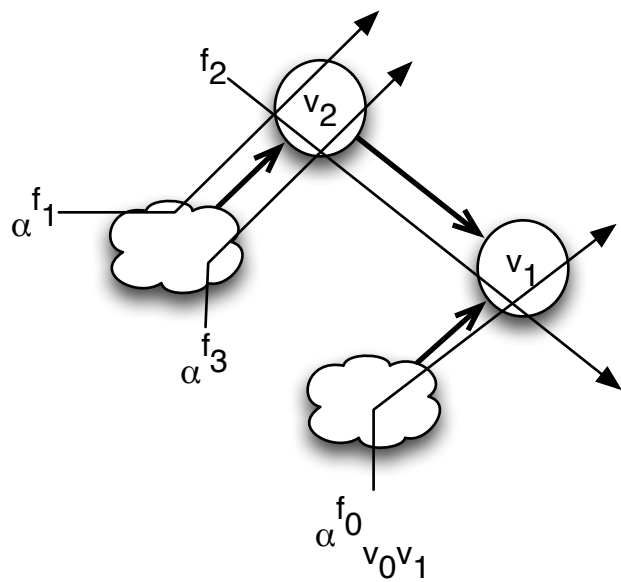
$$\begin{aligned} T_{v_0v_1}^{\text{PMOO l.o.}f_0} &= T_{v_0} + T_{v_1} + \frac{b_{v_0}^{xf_0} + b_{v_1}^{xf_0} + r_{v_0}^{xf_0} \cdot T_{v_0} + r_{v_1}^{xf_0} \cdot T_{v_1}}{(R_{v_0} - r_{v_0}^{xf_0}) \wedge (R_{v_1} - r_{v_1}^{xf_0})} \\ &= 20 + 20 + \frac{250 + 250 + 5 \cdot 20 + 5 \cdot 20}{15} \\ &= 86\frac{2}{3} \end{aligned}$$

$$\alpha_{v_1 v_3}^{f_0} = \gamma_{5,25} \oslash \beta_{15,86\frac{2}{3}} = \gamma_{5,458\frac{1}{3}} \beta^{\text{PMOO l.o.}f_1}.$$

$$\begin{aligned} R^{\text{PMOO l.o.}f_1} &= (R_{v_2} - r_{v_2}^{xf_1}) \wedge (R_{v_3} - r_{v_3}^{xf_1}) \\ &= (20 - 10) \wedge (20 - 10) \\ &= 10 \end{aligned}$$

$$\begin{aligned} T^{\text{PMOO l.o.}f_1} &= T_{v_2} + T_{v_3} + \frac{b_{v_2}^{xf_1} + b_{v_3}^{xf_1} + r_{v_2}^{xf_1} \cdot T_{v_2} + r_{v_3}^{xf_1} \cdot T_{v_3}}{(R_{v_2} - r_{v_2}^{xf_3}) \wedge (R_{v_3} - r_{v_3}^{xf_3})} \\ &= 20 + 20 + \frac{50 + 870\frac{5}{6} + 10 \cdot 20 + 10 \cdot 20}{10} \\ &= 40 + \frac{1320\frac{5}{6}}{10} \\ &= 172\frac{1}{12} \end{aligned}$$

$$\begin{aligned} D &= h(\alpha^{f_1}, \beta^{\text{PMOO l.o.}f_1}) = \frac{25}{10} + 172\frac{1}{12} = 174\frac{7}{12} \\ B &= v(\alpha^{f_1}, \beta^{\text{PMOO l.o.}f_1}) = 5 \cdot 172\frac{1}{12} + 25 = 885\frac{5}{12} \end{aligned}$$



TFA		FIFO_MUX	ARB_MUX
v_2	$\alpha_{v_1} = \alpha^{f_2} + \alpha^{f_1} + \alpha^{f_3}$	$= 3 \cdot \gamma_{5,25} = \gamma_{15,75}$	
	$D_{v_2}^{f_2}$	$\beta_{v_2} = b_{v_2}$ $20 \cdot [t - 20]^+ = 75$ $t = 23\frac{3}{4}$	$\beta_{v_2} = \alpha_{v_2}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 75$ $t = 95$
	$B_{v_2}^{f_2}$	$\alpha_{v_2}(T_{v_2}) = 15 \cdot 20 + 75$ $= 375$	
v_1	$\alpha_{v_1} = (\alpha_{v_2}^{f_2})^* + (\alpha_{v_0}^{f_0})^*$	$= \gamma_{5,137\frac{1}{2}} + \gamma_{5,159\frac{3}{8}} = \gamma_{10,296\frac{7}{8}}$	$= \gamma_{5,250} + \gamma_{5,241\frac{2}{3}} = \gamma_{10,491\frac{2}{3}}$
	$D_{v_1}^{f_2}$	$\beta_{v_1} = b_{v_1}$ $20 \cdot [t - 20]^+ = 296\frac{7}{8}$ $t = 34\frac{27}{32}$	$\beta_{v_1} = \alpha_{v_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 491\frac{2}{3}$ $t = 89\frac{1}{6}$
	$B_{v_1}^{f_2}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 296\frac{7}{8}$ $= 496\frac{7}{8}$	$\alpha_{v_1}(T_{v_1}) = 10 \cdot 20 + 491\frac{2}{3}$ $= 691\frac{2}{3}$
D^{f_2}		$D_{v_2}^{f_2} + D_{v_1}^{f_2} = 58\frac{19}{32}$	$D_{v_2}^{f_2} + D_{v_1}^{f_2} = 184\frac{1}{6}$
B^{f_2}		$\max\{B_{v_2}^{f_2}, B_{v_1}^{f_2}\} = 496\frac{7}{8}$	$\max\{B_{v_2}^{f_2}, B_{v_1}^{f_2}\} = 691\frac{2}{3}$

SFA Output Bound differing from TFA one:

computeOutputBound(v_2, f_3) = $(\alpha_{v_2}^{f_3})^*$		FIFO_MUX	ARB_MUX
$\alpha_{v_2}^{xf_3}$		$= \gamma_{5,25}$	
$\beta_{v_2}^{l.o.f_3} = [\beta_{v_2} - \alpha_{v_2}^{xf_3}]^+ = \beta_{R_{v_2}^{l.o.f_3}, T_{v_2}^{l.o.f_3}}$	$R_{v_2}^{l.o.f_3}$	$[R_{v_2} - r_{v_2}^{xf_3}]^+ = 15$	
	$T_{v_2}^{l.o.f_3}$	$\beta_{v_2} = b_{v_2}^{xf_3}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{v_2} = \alpha_{v_2}^{xf_3}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{10,21\frac{1}{4}}$	$= \beta_{10,28\frac{1}{3}}$
	$(r_{v_2}^{f_3})^*$	$= 5$	
$(\alpha_{v_2}^{f_3})^* = \alpha^{f_3} \odot \beta_{v_2}^{l.o.f_3} = \gamma_{(r_{v_2}^{f_3})^*, (b_{v_2}^{f_3})^*}$	$(b_{v_2}^{f_3})^*$	$\alpha^{f_3}(T_{v_2}^{l.o.f_3}) = 5 \cdot 21\frac{1}{4} + 25 = 126\frac{1}{4}$	$\alpha^{f_3}(T_{v_2}^{l.o.f_3}) = 5 \cdot 28\frac{1}{3} + 25 = 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
computeOutputBound(v_0, f_0) = $(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{xf_0} = \alpha_{v_2 v_0}^{f_3} = (\alpha_{v_2}^{f_3})^*$		$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$\beta_{v_0}^{l.o.f_0} = [\beta_{v_0} - \alpha_{v_0}^{xf_0}]^+ = \beta_{R_{v_0}^{l.o.f_0}, T_{v_0}^{l.o.f_0}}$	$R_{v_0}^{l.o.f_0}$	$[R_{v_0} - r_{v_0}^{xf_0}]^+ = 15$	
	$T_{v_0}^{l.o.f_0}$	$\beta_{v_0} = b_{v_0}^{xf_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{v_0} = \alpha_{v_0}^{xf_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{9}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{9}}$
	$r_{v_0 v_1}^{f_0}$	$= 5$	
$(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0} = \alpha^{f_0} \odot \beta_{v_0}^{l.o.f_0} = \gamma_{r_{v_0 v_1}^{f_0}, b_{v_0 v_1}^{f_0}}$	$b_{v_0 v_1}^{f_0}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 26\frac{9}{16} + 25 = 157\frac{13}{16}$	$\alpha^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 37\frac{7}{9} + 25 = 213\frac{8}{9}$
	$=$	$= \gamma_{5,157\frac{13}{16}}$	$= \gamma_{5,213\frac{8}{9}}$

SFA		FIFO_MUX	ARB_MUX
v_2	$\alpha_{v_2}^{xf_2} = \alpha^{f_1} + \alpha^{f_3}$	$= 2 \cdot \gamma_{2,25} = \gamma_{10,50}$	
	$\beta_{v_2}^{l.o.f_2} = [\beta_{v_2} - \alpha_{v_2}^{xf_2}]^+$	$= \beta_{10,22\frac{1}{2}}$	$= \beta_{10,45}$
v_1	$\alpha_{v_1}^{xf_2} = (\alpha_{v_0}^{f_0})^*$	$= \gamma_{5,157\frac{13}{16}}$	$= \gamma_{5,213\frac{8}{9}}$
	$\beta_{v_1}^{l.o.f_2} = [\beta_{v_1} - \alpha_{v_1}^{xf_2}]^+ = \beta_{R_{v_1}^{l.o.f_2}, T_{v_1}^{l.o.f_2}}$	$R_{v_1}^{l.o.f_2}$	$[R_{v_2} - r_{v_2}^{xf_2}]^+ = 15$
		$\beta_{v_1} = b_{v_1}^{xf_2}$	$\beta_{v_1} = \alpha_{v_1}^{xf_2}$
		$20 \cdot [t - 20]^+ = 157\frac{13}{16}$	$20 \cdot [t - 20]^+ = 5 \cdot t + 213\frac{8}{9}$
		$t = 27\frac{57}{64}$	$t = 40\frac{25}{27}$
	$=$	$= \beta_{10,27\frac{57}{64}}$	$= \beta_{10,40\frac{25}{27}}$
$\beta_{e2e}^{l.o.f_2} = \beta_{R_{e2e}^{l.o.f_2}, T_{e2e}^{l.o.f_2}}$		$\bigotimes_{i=1}^2 \beta_{v_i}^{l.o.f_2} = \beta_{10,50\frac{25}{64}}$	$\bigotimes_{i=1}^2 \beta_{v_i}^{l.o.f_2} = \beta_{10,85\frac{25}{27}}$
D^{f_2}		$\beta_{e2e}^{l.o.f_2} = b^{f_2}$	$\beta_{e2e}^{l.o.f_2} = b^{f_2}$
		$10 \cdot [t - 50\frac{25}{64}]^+ = 25$	$10 \cdot [t - 85\frac{25}{27}]^+ = 25$
		$t = 52\frac{57}{64}$	$t = 88\frac{23}{54}$
B^{f_2}		$\alpha^{f_2}(T_{e2e}^{l.o.f_2}) = 5 \cdot 50\frac{25}{64} + 25$	$\alpha^{f_2}(T_{e2e}^{l.o.f_2}) = 5 \cdot 85\frac{25}{27} + 25$
		$= 276\frac{61}{64}$	$= 454\frac{17}{27}$

PMOO (See [1] for details)

Cross traffic at node v_2 :

$$\alpha_{v_2}^{xf_2} = \alpha^{f_1} + \alpha^{f_3} = \gamma_{10,50}$$

Cross traffic at node v_1 :

$$\begin{aligned} \alpha_{v_1}^{xf_2} &= \alpha_{v_0 v_1}^{f_0} \\ &= \alpha^{f_0} \oslash \beta_{v_0}^{\text{PMOO l.o.}f_0} \\ &= \alpha^{f_0} \oslash [\beta_{v_0} - \alpha_{v_2 v_0}^{f_3}]^+ \\ &= \gamma_{5,25} \oslash [\beta_{20,20} - \gamma_{5,250}]^+ \\ &= \gamma_{5,25} \oslash \beta_{15,43\frac{1}{3}} \\ &= \gamma_{5,241\frac{2}{3}} \end{aligned}$$

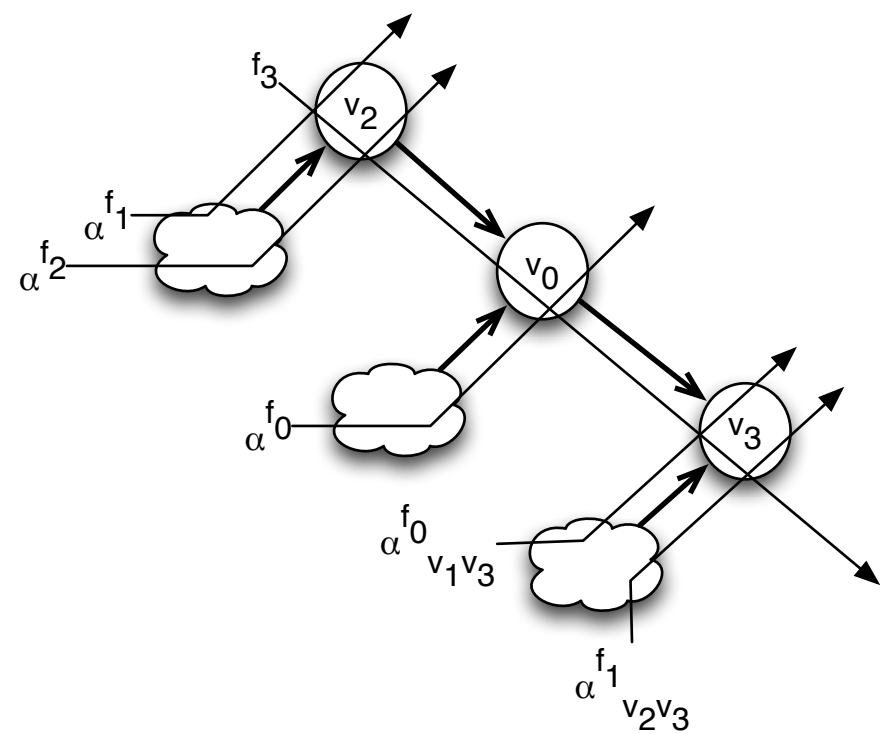
$\beta^{\text{PMOO l.o.}f_2}$:

$$\begin{aligned} R^{\text{PMOO l.o.}f_2} &= (R_{v_2} - r_{v_2}^{xf_2}) \wedge (R_{v_1} - r_{v_1}^{xf_2}) \\ &= (20 - 10) \wedge (20 - 5) \\ &= 10 \end{aligned}$$

$$\begin{aligned} T^{\text{PMOO l.o.}f_2} &= T_{v_2} + T_{v_1} + \frac{b_{v_2}^{xf_2} + b_{v_1}^{xf_2} + r_{v_2}^{xf_2} \cdot T_{v_2} + r_{v_1}^{xf_2} \cdot T_{v_1}}{(R_{v_2} - r_{v_2}^{xf_2}) \wedge (R_{v_1} - r_{v_1}^{xf_2})} \\ &= 20 + 20 + \frac{50 + 241\frac{2}{3} + 10 \cdot 20 + 5 \cdot 20}{10} \\ &= 99\frac{1}{6} \end{aligned}$$

$$\begin{aligned} D &= h(\alpha^{f_2}, \beta^{\text{PMOO l.o.}f_2}) = \frac{25}{10} + 99\frac{1}{6} = 101\frac{2}{3} \\ B &= v(\alpha^{f_2}, \beta^{\text{PMOO l.o.}f_2}) = 5 \cdot 99\frac{1}{6} + 25 = 520\frac{5}{6} \end{aligned}$$

Flow f_3



TFA		FIFO_MUX	ARB_MUX
v_2	$\alpha_{v_2} = \alpha^{f_3} + \alpha^{f_1} + \alpha^{f_2}$	$= 3 \cdot \gamma_{5,25} = \gamma_{15,75}$	
	$D_{v_2}^{f_3}$	$\beta_{v_2} = b_{v_2}$ $20 \cdot [t - 20]^+ = 75$ $t = 23\frac{3}{4}$	$\beta_{v_2} = \alpha_{v_2}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 75$ $t = 95$
	$B_{v_2}^{f_3}$	$\alpha_{v_2}(T_{v_2}) = 15 \cdot 20 + 75$ $= 375$	
v_0	$\alpha_{v_0} = \alpha^{f_0} + \alpha_{v_2 v_0}^{f_3} = \alpha^{f_0} + (\alpha_{v_2}^{f_3})^*$	$= \gamma_{5,25} + \gamma_{5,137\frac{1}{2}} = \gamma_{10,162\frac{1}{2}}$	$= \gamma_{5,25} + \gamma_{5,250} = \gamma_{10,275}$
	$D_{v_0}^{f_3}$	$\beta_{v_0} = b_{v_0}$ $20 \cdot [t - 20]^+ = 162\frac{1}{2}$ $t = 28\frac{1}{8}$	$\beta_{v_0} = \alpha_{v_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 275$ $t = 67\frac{1}{2}$
	$B_{v_0}^{f_3}$	$\alpha_{v_0}(T_{v_0}) = 10 \cdot 20 + 162\frac{1}{2}$ $= 262\frac{1}{2}$	$\alpha_{v_0}(T_{v_0}) = 10 \cdot 20 + 275$ $= 475$
v_3	$\alpha_{v_3} = (\alpha_{v_1}^{f_0})^* + (\alpha_{v_2}^{f_1})^* + (\alpha_{v_0}^{f_3})^*$	$= \gamma_{5,293\frac{3}{4}} + \gamma_{5,137\frac{1}{2}} + \gamma_{5,243\frac{3}{4}} = \gamma_{15,675}$	$= \gamma_{5,458\frac{1}{3}} + \gamma_{5,250} + \gamma_{5,391\frac{2}{3}} = \gamma_{15,1100}$
	$D_{v_3}^{f_3}$	$\beta_{v_3} = b_{v_3}$ $20 \cdot [t - 20]^+ = 675$ $t = 53\frac{3}{4}$	$\beta_{v_3} = \alpha_{v_3}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 1100$ $t = 300$
	$B_{v_3}^{f_3}$	$\alpha_{v_3}(T_{v_3}) = 15 \cdot 20 + 675$ $= 975$	$\alpha_{v_3}(T_{v_3}) = 15 \cdot 20 + 1100$ $= 1400$
D^{f_3}		$D_{v_2}^{f_3} + D_{v_0}^{f_3} + D_{v_3}^{f_3} = 105\frac{5}{8}$	$D_{v_2}^{f_3} + D_{v_0}^{f_3} + D_{v_3}^{f_3} = 462\frac{1}{2}$
B^{f_3}		$\max\{B_{v_2}^{f_3}, B_{v_0}^{f_3}, B_{v_3}^{f_3}\} = 975$	$\max\{B_{v_2}^{f_3}, B_{v_0}^{f_3}, B_{v_3}^{f_3}\} = 1400$

SFA Output Bound differing from TFA one:

computeOutputBound(v_2, f_i) = $(\alpha_{v_2}^{f_i})^*$, $i \in \{1, 2\}$		FIFO_MUX	ARB_MUX
$\alpha_{v_2}^{xf_i} = \alpha_{v_2}^{xf_2} = \alpha_{v_2}^{xf_3}$		$= \gamma_{5,25}$	
$\beta_{v_2}^{l.o.f_i} = [\beta_{v_2} - \alpha_{v_2}^{xf_i}]^+ = \beta_{R_{v_2}^{l.o.f_i}, T_{v_2}^{l.o.f_i}}$	$R_{v_2}^{l.o.f_i}$	$[R_{v_2} - r_{v_2}^{xf_i}]^+ = 15$	
	$T_{v_2}^{l.o.f_i}$	$\beta_{v_2} = b_{v_2}^{xf_i}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{v_2} = \alpha_{v_2}^{xf_i}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{10,21\frac{1}{4}}$	$= \beta_{10,28\frac{1}{3}}$
	$(r_{v_2}^{f_i})^*$	$= 5$	
$(\alpha_{v_2}^{f_i})^* = \alpha_{v_2}^{f_i} \odot \beta_{v_2}^{l.o.f_i} = \gamma_{(r_{v_2}^{f_i})^*, (b_{v_2}^{f_i})^*}$	$(b_{v_2}^{f_i})^*$	$\alpha_{v_2}^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 21\frac{1}{4} + 25 = 126\frac{1}{4}$	$\alpha_{v_2}^{f_i}(T_{v_2}^{l.o.f_i}) = 5 \cdot 28\frac{1}{3} + 25 = 166\frac{2}{3}$
	$=$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
computeOutputBound(v_0, f_0) = $(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{xf_0}$		$= \gamma_{0,0}$	
$\beta_{v_0}^{l.o.f_0} = [\beta_{v_0} - \alpha_{v_0}^{xf_0}]^+$		$= \beta_{20,20}$	
$(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_1}^{f_0} = \alpha_{v_0}^{f_0} \odot \beta_{v_0}^{l.o.f_0} = \gamma_{r_{v_0 v_1}^{f_0}, b_{v_0 v_1}^{f_0}}$	$r_{v_0 v_1}^{f_0}$	$= 5$	
	$b_{v_0 v_1}^{f_0}$	$\alpha_{v_0}^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 20 + 25 = 125$	
	$=$	$= \gamma_{5,125}$	
computeOutputBound(v_1, f_0) = $(\alpha_{v_1}^{f_0})^* = \alpha_{v_1 v_3}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_1}^{xf_0} = \alpha_{v_2 v_1}^{f_2} = (\alpha_{v_2}^{f_2})^*$		$= \gamma_{5,131\frac{1}{4}}$	
$\beta_{v_1}^{l.o.f_0} = [\beta_{v_1} - \alpha_{v_1}^{xf_0}]^+ = \beta_{R_{v_1}^{l.o.f_0}, T_{v_1}^{l.o.f_0}}$	$R_{v_1}^{l.o.f_0}$	$[R_{v_1} - r_{v_1}^{xf_0}]^+ = 15$	
	$T_{v_1}^{l.o.f_0}$	$\beta_{v_0} = b_{v_0}^{xf_0}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	$\beta_{v_0} = \alpha_{v_0}^{xf_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 166\frac{2}{3}$ $t = 37\frac{7}{9}$
	$=$	$= \beta_{15,26\frac{9}{16}}$	$= \beta_{15,37\frac{7}{9}}$
	$r_{v_1 v_3}^{f_0}$	$= 5$	
$(\alpha_{v_1}^{f_0})^* = (\alpha_{v_0}^{f_0})^* \odot \beta_{v_1}^{l.o.f_0} = \gamma_{r_{v_1 v_3}^{f_0}, b_{v_1 v_3}^{f_0}}$	$b_{v_1 v_3}^{f_0}$	$\alpha_{v_0}^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 26\frac{9}{16} + 125 = 257\frac{13}{16}$	$\alpha_{v_0}^{f_0}(T_{v_0}^{l.o.f_0}) = 5 \cdot 37\frac{7}{9} + 125 = 313\frac{8}{9}$
	$=$	$= \gamma_{5,257\frac{13}{16}}$	$= \gamma_{5,313\frac{8}{9}}$

SFA			FIFO_MUX	ARB_MUX		
v_2	$\alpha_{v_2}^{xf_3} = \alpha^{f_1} + \alpha^{f_2}$		$= 2 \cdot \gamma_{2,25} = \gamma_{10,50}$			
	$\beta_{v_2}^{l.o.f_3} = [\beta_{v_2} - \alpha_{v_2}^{xf_3}]^+$		$= \beta_{10,22\frac{1}{2}}$	$= \beta_{10,45}$		
v_0	$\alpha_{v_0}^{xf_3} = \alpha^{f_0}$		$= \gamma_{5,25}$			
	$\beta_{v_0}^{l.o.f_3} = [\beta_{v_0} - \alpha_{v_0}^{xf_3}]^+$		$= \beta_{15,21\frac{1}{4}}$	$= \beta_{15,28\frac{1}{3}}$		
v_3	$\alpha_{v_3}^{xf_3} = (\alpha_{v_1}^{f_0})^* + (\alpha_{v_2}^{f_1})^*$		$= \gamma_{5,257\frac{13}{16}} + \gamma_{5,131\frac{1}{4}} = \gamma_{10,389\frac{1}{16}}$	$= \gamma_{5,313\frac{8}{9}} + \gamma_{5,166\frac{2}{3}} = \gamma_{10,480\frac{5}{9}}$		
	$\beta_{v_3}^{l.o.f_3} = [\beta_{v_3} - \alpha_{v_3}^{xf_3}]^+ = \beta_{R_{v_3}^{l.o.f_3}, T_{v_3}^{l.o.f_3}}$	$R_{v_3}^{l.o.f_3}$	$[R_{v_3} - r_{v_3}^{xf_3}]^+ = 10$			
		$T_{v_3}^{l.o.f_3}$	$\beta_{v_3} =$	$\beta_{v_3} =$	$\alpha_{v_3}^{xf_3}$	
			$20 \cdot [t - 20]^+ =$	$389\frac{1}{16}$	$20 \cdot [t - 20]^+ =$	$10 \cdot t + 480\frac{5}{9}$
			$t =$	$39\frac{29}{64}$	$t =$	$88\frac{1}{18}$
	$=$	$= \beta_{10,39\frac{29}{64}}$	$= \beta_{10,88\frac{1}{18}}$			
$\beta_{e2e}^{l.o.f_3} = \beta_{R_{e2e}^{l.o.f_3}, T_{e2e}^{l.o.f_3}}$			$\bigotimes_{i=\{2,0,3\}} \beta_{v_i}^{l.o.f_3} = \beta_{10,83\frac{13}{64}}$	$\bigotimes_{i=\{2,0,3\}} \beta_{v_i}^{l.o.f_3} = \beta_{10,161\frac{7}{18}}$		
D^{f_3}			$\beta_{e2e}^{l.o.f_3} =$	$\beta_{e2e}^{l.o.f_3} =$	b^{f_3}	
			$10 \cdot [t - 83\frac{13}{64}]^+ =$	25	$10 \cdot [t - 161\frac{7}{18}]^+ =$	25
			$t =$	$85\frac{45}{64}$	$t =$	$163\frac{8}{9}$
B^{f_3}			$\alpha^{f_3}(T_{e2e}^{l.o.f_3}) =$	$5 \cdot 83\frac{13}{64} + 25$	$\alpha^{f_3}(T_{e2e}^{l.o.f_3}) =$	$5 \cdot 161\frac{7}{18} + 25$
			$=$	$441\frac{1}{64}$	$=$	$831\frac{17}{18}$

PMOO (See [1] for details)

Cross traffic at node v_2 :

$$\alpha_{v_0}^{xf_3} = \alpha^{f_1} + \alpha^{f_2} = \gamma_{10,50}$$

Cross traffic at node v_0 :

$$\alpha_{v_0}^{xf_3} = \alpha^{f_0} = \gamma_{5,25}$$

Cross traffic at node v_3 :

$$\alpha_{v_3}^{xf_3} = \alpha_{v_1v_3}^{f_0} + \alpha_{v_2v_3}^{f_1} = \gamma_{10,708\frac{1}{3}} \text{ with}$$

$$\alpha_{v_2v_3}^{f_1} = \gamma_{5,250}$$

(see $\text{computeOutputBound}(v_2, f_i) = (\alpha_{v_2}^{f_i})^*$, $i \in \{1, 2, 3\}$ above because f_1 only crosses a single server),

$$\alpha_{v_1v_3}^{f_0} = \gamma_{5,458\frac{1}{3}}$$

(see f_1)

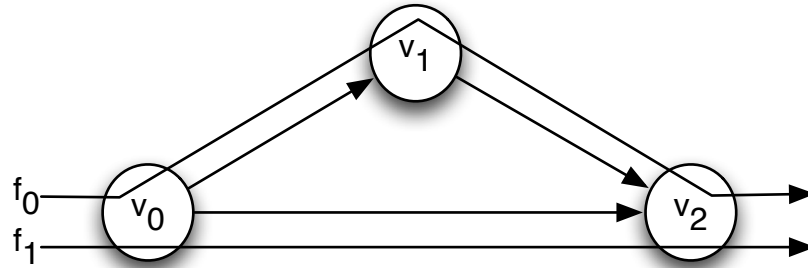
$$\beta^{\text{PMOO l.o. } f_3}:$$

$$\begin{aligned} R^{\text{PMOO l.o. } f_3} &= (R_{v_2} - r_{v_2}^{xf_3}) \wedge (R_{v_0} - r_{v_0}^{xf_3}) \wedge (R_{v_3} - r_{v_3}^{xf_3}) \\ &= (20 - 10) \wedge (20 - 5) \wedge (20 - 10) \\ &= 10 \end{aligned}$$

$$\begin{aligned} T^{\text{PMOO l.o. } f_3} &= T_{v_2} + T_{v_0} + T_{v_3} + \frac{b_{v_2}^{xf_3} + b_{v_0}^{xf_3} + b_{v_3}^{xf_3} + r_{v_2}^{xf_3} \cdot T_{v_2} + r_{v_0}^{xf_3} \cdot T_{v_0} + r_{v_3}^{xf_3} \cdot T_{v_3}}{(R_{v_2} - r_{v_2}^{xf_3}) \wedge (R_{v_0} - r_{v_0}^{xf_3}) \wedge (R_{v_3} - r_{v_3}^{xf_3})} \\ &= 20 + 20 + 20 + \frac{50 + 25 + 708\frac{1}{3} + 10 \cdot 20 + 5 \cdot 20 + 10 \cdot 20}{10} \\ &= 60 + \frac{1283\frac{1}{3}}{10} \\ &= 188\frac{1}{3} \end{aligned}$$

$$\begin{aligned} D &= h(\alpha^{f_3}, \beta^{\text{PMOO l.o. } f_3}) = \frac{25}{10} + 188\frac{1}{3} = 190\frac{5}{6} \\ B &= v(\alpha^{f_3}, \beta^{\text{PMOO l.o. } f_3}) = 5 \cdot 188\frac{1}{3} + 25 = 966\frac{2}{3} \end{aligned}$$

FeedForward_1SC_2Flows_1AC_2Paths



(naming of flows is switched in the following)

- $\beta_{v_0} = \beta_{v_1} = \beta_{v_2} = \beta_{R_{v_i}, T_{v_i}} = \beta_{20,20}, i \in \{0, 1, 3\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_j}, b^{f_j}} = \gamma_{5,25}, j \in \{0, 1, 2\}$

$\text{computeOutputBound}(v_0, f_0) = (\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_2}^{f_0}$		FIFO_MUX	ARB_MUX
$\alpha_{v_0}^{x f_0} = \alpha^{f_1}$		$= \gamma_{5,25}$	
$\beta_{v_0}^{\text{l.o.} f_0} = [\beta_{v_0} - \alpha_{v_0}^{x f_0}]^+$	$R_{v_0}^{\text{l.o.} f_0}$	$= 15$	
	$T_{v_0}^{\text{l.o.} f_0}$	$\beta_{v_0} = b_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	$\beta_{v_0} = \alpha_{v_0}^{x f_0}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 25$ $t = 28\frac{1}{3}$
	$=$	$= \beta_{15, 21\frac{1}{4}}$	$= \beta_{15, 28\frac{1}{3}}$
	$=$	$= 5$	$= 5$
$\alpha_{v_0 v_2}^{f_0} = \alpha^{f_0} \oslash \beta_{v_0}^{\text{l.o.} f_0}$	$r_{v_0 v_2}^{f_0}$	$\alpha^{f_0}(T_{v_0}^{\text{l.o.} f_0}) = 131\frac{1}{4}$	
	$b_{v_0 v_2}^{f_0}$	$\alpha^{f_0}(T_{v_0}^{\text{l.o.} f_0}) = 166\frac{2}{3}$	
	$=$	$= \gamma_{5, 131\frac{1}{4}}$	
$\text{computeOutputBound}(v_0, f_1) = (\alpha_{v_0}^{f_1})^* = \alpha_{v_0 v_1}^{f_1}$		FIFO_MUX	ARB_MUX
$= \text{computeOutputBound}(v_0, f_0) = (\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_2}^{f_0}$		$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$
$\text{computeOutputBound}(v_1, f_1) = (\alpha_{v_1}^{f_1})^* = \alpha_{v_1 v_2}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{v_1} = \alpha_{v_0 v_1}^{f_1}$		$= \gamma_{5, 131\frac{1}{4}}$	$= \gamma_{5, 166\frac{2}{3}}$
$\alpha_{v_1}^{x f_1}$		$= \gamma_{0,0}$	
$\beta_{v_1}^{\text{l.o.} f_1} = [\beta_{v_1} - \alpha_{v_1}^{x f_1}]^+$		$= \beta_{20,20}$	
$\alpha_{v_1 v_2}^{f_1} = \alpha_{v_1}^{f_1} \oslash \beta_{v_1}^{\text{l.o.} f_1} = \alpha^{f_1} \oslash \beta_{v_1}^{\text{l.o.} f_1}$	$r_{v_1 v_2}^{f_1}$	$= 5$	
	$b_{v_1 v_2}^{f_1}$	$\alpha^{f_0}(T_{v_0}^{\text{l.o.} f_0}) = 231\frac{1}{4}$	$\alpha^{f_0}(T_{v_0}^{\text{l.o.} f_0}) = 266\frac{2}{3}$
	$=$	$= \gamma_{5, 231\frac{1}{4}}$	$= \gamma_{5, 266\frac{2}{3}}$

computeFifoOutputBound(v_0, f_0) = $(\alpha_{v_0}^{f_0})^* = \alpha_{v_0 v_2}^{f_0}$		FIFO_MUX	ARB_MUX
flow of interest f_1			
$\alpha_{v_0}^{f_0} = \alpha^{f_0}$		$= \gamma_{5,25}$	
$\alpha_{v_0}^{x f_0}$		$= \gamma_{0,0}$	
$\beta_{v_0}^{\text{SFA l.o. } f_0} = [\beta_{v_0} - \alpha_{v_0}^{x f_0}]^+$		$= \beta_{20,20}$	
$\alpha_{v_0 v_2}^{f_0} = \alpha^{f_0} \oslash \beta_{v_0}^{\text{l.o. } f_0}$	$r_{v_0 v_2}^{f_0}$	$= 5$	
	$b_{v_0 v_2}^{f_0}$	$\alpha^{f_0}(T_{v_0}^{\text{l.o. } f_0}) = 125$	
	$=$	$= \gamma_{5,125}$	

Flow f_0

TFA		FIFO _ MUX	ARB _ MUX
v_0	$\alpha_{v_1} = \alpha_{v_0}^{f_0} + \alpha_{v_1}^{f_1}$	$= \gamma_{10,50}$	
	$D_{v_0}^{f_0}$	$\beta_{v_0} = b_{v_0}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{v_0} = \alpha_{v_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{v_0}^{f_0}$	$\alpha_{v_0}(T_{v_0}) = 10 \cdot 20 + 50$ $= 250$	
$v_0 v_2$	$\alpha_{v_0 v_2} = \alpha_{v_0 v_2}^{f_0}$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$v_1 v_2$	$\alpha_{v_1 v_2} = \alpha_{v_1 v_2}^{f_1}$	$= \gamma_{5,231\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}}$
v_2	$\alpha_{v_2} = \alpha_{v_0 v_2} + \alpha_{v_1 v_2}$	$= \gamma_{10,362\frac{1}{2}}$	
	$D_{v_2}^{f_0}$	$\beta_{v_2} = b_{v_2}$ $20 \cdot [t - 20]^+ = 362\frac{1}{2}$ $t = 38\frac{1}{8}$	$\beta_{v_2} = \alpha_{v_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 433\frac{1}{3}$ $t = 83\frac{1}{3}$
	$B_{v_2}^{f_0}$	$\alpha_{v_2}(T_{v_2}) = 10 \cdot 20 + 362\frac{1}{2}$ $= 562\frac{1}{2}$	
D^{f_0}		$= 60\frac{5}{8}$	$= 128\frac{1}{3}$
B^{f_0}		$\max_{i=\{0,2\}} b_{v_i}^{f_0} = 562\frac{1}{2}$	$\max_{i=\{0,2\}} b_{v_i}^{f_0} = 633\frac{1}{3}$

SFA FIFO_MUX:

$$\begin{aligned}
\beta_{\text{e2e}}^{\text{SFA } f_0 \text{ l.o. } f_0} &= [\beta_{v_0}^{\text{SFA } f_0 \text{ l.o. } x f_0} - \alpha_{v_0}^{x f_0}]^+ \otimes [\beta_{v_2}^{\text{SFA } f_0 \text{ l.o. } x f_0} - \alpha_{v_2}^{x f_0}]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - \alpha_{v_1 v_2}^{f_1}]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - (\alpha_{v_1}^{f_1} \oslash \beta_{v_1}^{\text{SFA } f_0 \text{ l.o. } x f_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - ((\alpha^{f_1} \oslash \beta_{v_0}^{\text{SFA } f_0 \text{ l.o. } x f_1}) \oslash \beta_{v_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - ((\alpha^{f_1} \oslash \beta_{v_0}) \oslash \beta_{v_1})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes [\beta_{20,20} - ((\gamma_{5,25} \oslash \beta_{20,20}) \oslash \beta_{20,20})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes [\beta_{20,20} - (\gamma_{5,125} \oslash \beta_{20,20})]^+ \\
&= \beta_{15,21\frac{1}{4}} \otimes [\beta_{20,20} - \gamma_{5,225}]^+ \\
&= \beta_{15,21\frac{1}{4}} \otimes \beta_{15,31\frac{1}{4}} \\
&= \beta_{15,52\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{\text{e2e}}^{\text{l.o. } f_1} \cdot T_{\text{e2e}}^{\text{l.o. } f_1} + b^{f_1}}{R_{\text{e2e}}^{\text{l.o. } f_1}} \\
&= \frac{15 \cdot 52\frac{1}{2} + 25}{15} \\
&= 54\frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1}(T_{\text{e2e}}^{\text{l.o. } f_1}) \\
&= 5 \cdot 52\frac{1}{2} + 25 \\
&= 287\frac{1}{2}
\end{aligned}$$

SFA ARB_MUX:

$$\begin{aligned}
\beta_{\text{e2e}}^{\text{SFA } f_0 \text{ l.o. } f_0} &= [\beta_{v_0}^{\text{SFA } f_0 \text{ l.o. } x f_0} - \alpha_{v_0}^{x f_0}]^+ \otimes [\beta_{v_2}^{\text{SFA } f_0 \text{ l.o. } x f_0} - \alpha_{v_2}^{x f_0}]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - \alpha_{v_1 v_2}^{f_1}]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - (\alpha_{v_1}^{f_1} \otimes \beta_{v_1}^{\text{SFA } f_0 \text{ l.o. } x f_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - ((\alpha^{f_1} \otimes \beta_{v_0}^{\text{SFA } f_0 \text{ l.o. } x f_1}) \otimes \beta_{v_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - ((\alpha^{f_1} \otimes \beta_{v_0}) \otimes \beta_{v_1})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes [\beta_{20,20} - ((\gamma_{5,25} \otimes \beta_{20,20}) \otimes \beta_{20,20})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes [\beta_{20,20} - (\gamma_{5,125} \otimes \beta_{20,20})]^+ \\
&= \beta_{15,28\frac{1}{3}} \otimes [\beta_{20,20} - \gamma_{5,225}]^+ \\
&= \beta_{15,28\frac{1}{3}} \otimes \beta_{15,41\frac{2}{3}} \\
&= \beta_{15,70}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{\text{e2e}}^{\text{l.o. } f_1} \cdot T_{\text{e2e}}^{\text{l.o. } f_1} + b^{f_1}}{R_{\text{e2e}}^{\text{l.o. } f_1}} \\
&= \frac{15 \cdot 70 + 25}{15} \\
&= 71\frac{2}{3}
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1}(T_{\text{e2e}}^{\text{l.o. } f_1}) \\
&= 5 \cdot 70 + 25 \\
&= 375
\end{aligned}$$

PMOO ARB_MUX:

$$\begin{aligned}
\beta_{\text{e2e}}^{\text{l.o.},f_0} &= [\beta_{v_0}^{\text{SFA l.o.},xf_0} - \alpha_{v_0}^{xf_0}]^+ \otimes [\beta_{v_2}^{\text{SFA l.o.},xf_0} - \alpha_{v_2}^{xf_0}]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - \alpha_{v_1 v_2}^{f_1}]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - (\alpha_{v_1}^{f_1} \otimes \beta_{v_1}^{\text{l.o.},f_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - ((\alpha^{f_1} \otimes \beta_{v_0}^{\text{l.o.},f_1}) \otimes \beta_{v_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_1}]^+ \otimes [\beta_{v_2} - ((\alpha^{f_1} \otimes [\beta_{v_0} - \alpha^{f_0}]^+) \otimes \beta_{v_1})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes [\beta_{20,20} - ((\gamma_{5,25} \otimes [\beta_{20,20} - \gamma_{5,25}]^+) \otimes \beta_{20,20})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes [\beta_{20,20} - ((\gamma_{5,25} \otimes \beta_{20,28\frac{1}{3}}) \otimes \beta_{20,20})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes [\beta_{20,20} - (\gamma_{5,166\frac{2}{3}} \otimes \beta_{20,20})]^+ \\
&= \beta_{15,28\frac{1}{3}} \otimes [\beta_{20,20} - \gamma_{5,266\frac{2}{3}}]^+ \\
&= \beta_{15,28\frac{1}{3}} \otimes \beta_{15,44\frac{4}{9}} \\
&= \beta_{15,72\frac{7}{9}}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{\text{e2e}}^{\text{l.o.},f_1} \cdot T_{\text{e2e}}^{\text{l.o.},f_1} + b^{f_1}}{R_{\text{e2e}}^{\text{l.o.},f_1}} \\
&= \frac{15 \cdot 72\frac{7}{9} + 25}{15} \\
&= 74\frac{4}{9}
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1}(T_{\text{e2e}}^{\text{l.o.},f_1}) \\
&= 5 \cdot 72\frac{7}{9} + 25 \\
&= 388\frac{8}{9}
\end{aligned}$$

Flow f_1

TFA		FIFO_MUX	ARB_MUX
v_0	$\alpha_{v_1} = \alpha_{v_0}^{f_0} + \alpha_{v_1}^{f_1}$	$= \gamma_{10,50}$	
	$D_{v_0}^{f_0}$	$\beta_{v_0} = b_{v_0}$ $20 \cdot [t - 20]^+ = 50$ $t = 22\frac{1}{2}$	$\beta_{v_0} = \alpha_{v_0}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 50$ $t = 45$
	$B_{v_0}^{f_0}$	$\alpha_{v_0}(T_{v_0}) = 10 \cdot 20 + 50$ $= 250$	
$v_0 v_1$	$\alpha_{v_0 v_1} = \alpha_{v_0 v_1}^{f_1}$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
v_1	$\alpha_{v_1} = \alpha_{v_0 v_1}^{f_1}$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
	$D_{v_1}^{f_1}$	$\beta_{v_1} = b_{v_1}$ $20 \cdot [t - 20]^+ = 131\frac{1}{4}$ $t = 26\frac{9}{16}$	FIFO per micro flow $\beta_{v_1} = b_{v_1}$ $20 \cdot [t - 20]^+ = 166\frac{2}{3}$ $t = 28\frac{1}{3}$
	$B_{v_1}^{f_1}$	$\alpha_{v_1}(T_{v_1}) = 5 \cdot 20 + 131\frac{1}{4}$ $= 231\frac{1}{4}$	$\alpha_{v_1}(T_{v_1}) = 5 \cdot 20 + 166\frac{2}{3}$ $= 266\frac{2}{3}$
$v_0 v_2$	$\alpha_{v_0 v_2} = \alpha_{v_0 v_2}^{f_0}$	$= \gamma_{5,131\frac{1}{4}}$	$= \gamma_{5,166\frac{2}{3}}$
$v_1 v_2$	$\alpha_{v_1 v_2} = \alpha_{v_1 v_2}^{f_1}$	$= \gamma_{5,231\frac{1}{4}}$	$= \gamma_{5,266\frac{2}{3}}$
v_2	$\alpha_{v_2} = \alpha_{v_0 v_2} + \alpha_{v_1 v_2}$	$= \gamma_{10,362\frac{1}{2}}$	$= \gamma_{10,433\frac{1}{3}}$
	$D_{v_2}^{f_1}$	$\beta_{v_2} = b_{v_2}$ $20 \cdot [t - 20]^+ = 362\frac{1}{2}$ $t = 38\frac{1}{8}$	$\beta_{v_2} = \alpha_{v_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 433\frac{1}{3}$ $t = 83\frac{1}{3}$
	$B_{v_2}^{f_1}$	$\alpha_{v_2}(T_{v_2}) = 10 \cdot 20 + 362\frac{1}{2}$ $= 562\frac{1}{2}$	$\alpha_{v_2}(T_{v_2}) = 10 \cdot 20 + 433\frac{1}{3}$ $= 633\frac{1}{3}$
D^{f_1}		$\sum_{i=0}^2 \beta_{v_i}^{f_1} = 87\frac{3}{16}$	$\sum_{i=0}^2 \beta_{v_i}^{f_1} = 156\frac{2}{3}$
B^{f_1}		$\max_{i=0}^2 b_{v_i}^{f_1} = 562\frac{1}{2}$	$\max_{i=0}^2 b_{v_i}^{f_1} = 633\frac{1}{3}$

SFA FIFO_MUX:

$$\begin{aligned}
\beta_{\mathbf{e}2\mathbf{e}}^{\text{SFA } f_1 \text{ l.o. } f_1} &= [\beta_{v_0}^{\text{SFA } f_1 \text{ l.o. } x f_1} - \alpha_{v_0}^{x f_1}]^+ \otimes [\beta_{v_1}^{\text{SFA } f_1 \text{ l.o. } x f_1} - \alpha_{v_1}^{x f_1}]^+ \otimes [\beta_{v_2}^{\text{SFA } f_1 \text{ l.o. } x f_1} - \alpha_{v_2}^{x f_1}]^+ \\
&= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - \alpha_{v_0 v_2}^{f_0}]^+ \\
&= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - (\alpha_{v_0}^{f_0} \oslash \beta_{v_0}^{\text{SFA } f_1 \text{ l.o. } x f_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - (\alpha^{f_0} \oslash \beta_{v_0})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes \beta_{20,20} \otimes [\beta_{20,20} - (\gamma_{5,25} \oslash \beta_{20,20})]^+ \\
&= \beta_{15,21\frac{1}{4}} \otimes \beta_{20,20} \otimes [\beta_{20,20} - \gamma_{5,125}]^+ \\
&= \beta_{15,21\frac{1}{4}} \otimes \beta_{20,20} \otimes \beta_{15,26\frac{1}{4}} \\
&= \beta_{15,67\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{\mathbf{e}2\mathbf{e}}^{\text{l.o. } f_1} \cdot T_{\mathbf{e}2\mathbf{e}}^{\text{l.o. } f_1} + b^{f_1}}{R_{\mathbf{e}2\mathbf{e}}^{\text{l.o. } f_1}} \\
&= \frac{15 \cdot 67\frac{1}{2} + 25}{15} \\
&= 69\frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1}(T_{\mathbf{e}2\mathbf{e}}^{\text{l.o. } f_1}) \\
&= 5 \cdot 67\frac{1}{2} + 25 \\
&= 362\frac{1}{2}
\end{aligned}$$

SFA ARB_MUX:

$$\begin{aligned}
\beta_{\text{e2e}}^{\text{SFA } f_1 \text{ l.o.} f_1} &= [\beta_{v_0}^{\text{SFA } f_1 \text{ l.o.} x f_1} - \alpha_{v_0}^{x f_1}]^+ \otimes [\beta_{v_1}^{\text{SFA } f_1 \text{ l.o.} x f_1} - \alpha_{v_1}^{x f_1}]^+ \otimes [\beta_{v_2}^{\text{SFA } f_1 \text{ l.o.} x f_1} - \alpha_{v_2}^{x f_1}]^+ \\
&= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - \alpha_{v_0 v_2}^{f_0}]^+ \\
&= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - (\alpha_{v_0}^{f_0} \oslash \beta_{v_0}^{\text{SFA } f_1 \text{ l.o.} x f_1})]^+ \\
&= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - (\alpha^{f_0} \oslash \beta_{v_0})]^+ \\
&= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes \beta_{20,20} \otimes [\beta_{20,20} - (\gamma_{5,25} \oslash \beta_{20,20})]^+ \\
&= \beta_{15,28\frac{1}{3}} \otimes \beta_{20,20} \otimes [\beta_{20,20} - \gamma_{5,125}]^+ \\
&= \beta_{15,28\frac{1}{3}} \otimes \beta_{20,20} \otimes \beta_{15,35} \\
&= \beta_{15,83\frac{1}{3}}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{\text{e2e}}^{\text{l.o.} f_1} \cdot T_{\text{e2e}}^{\text{l.o.} f_1} + b^{f_1}}{R_{\text{e2e}}^{\text{l.o.} f_1}} \\
&= \frac{15 \cdot 83\frac{1}{3} + 25}{15} \\
&= 85
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1}(T_{\text{e2e}}^{\text{l.o.} f_1}) \\
&= 5 \cdot 83\frac{1}{3} + 25 \\
&= 441\frac{2}{3}
\end{aligned}$$

PMOO ARB_MUX:

$$\beta_{\text{e2e}}^{\text{PMOO l.o. } f_1} =$$

$$=$$

$$\begin{aligned} \beta_{\text{e2e}}^{\text{l.o. } f_1} &= [\beta_{v_0}^{\text{SFA l.o. } x f_1} - \alpha_{v_0}^{x f_1}]^+ \otimes [\beta_{v_1}^{\text{SFA l.o. } x f_1} - \alpha_{v_1}^{x f_1}]^+ \otimes [\beta_{v_2}^{\text{SFA l.o. } x f_1} - \alpha_{v_2}^{x f_1}]^+ \\ &= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - \alpha_{v_0 v_2}^{f_0}]^+ \\ &= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - (\alpha_{v_0}^{f_0} \oslash \beta_{v_0}^{\text{l.o. } x f_1})]^+ \\ &= [\beta_{v_0} - \alpha^{f_0}]^+ \otimes \beta_{v_1} \otimes [\beta_{v_2} - (\alpha^{f_0} \oslash [\beta_{v_0} - \alpha^{f_1}]^+)]^+ \\ &= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes \beta_{20,20} \otimes [\beta_{20,20} - (\gamma_{5,25} \oslash [\beta_{20,20} - \gamma_{5,25}]^+)]^+ \\ &= [\beta_{20,20} - \gamma_{5,25}]^+ \otimes \beta_{20,20} \otimes [\beta_{20,20} - (\gamma_{5,25} \oslash \beta_{15,28\frac{1}{3}})]^+ \\ &= \beta_{15,28\frac{1}{3}} \otimes \beta_{20,20} \otimes [\beta_{20,20} - \gamma_{5,166\frac{2}{3}}]^+ \\ &= \beta_{15,28\frac{1}{3}} \otimes \beta_{20,20} \otimes \beta_{15,37\frac{7}{9}} \\ &= \beta_{15,86\frac{1}{9}} \end{aligned}$$

$$\begin{aligned} D^{f_1} &= \frac{R_{\text{e2e}}^{\text{l.o. } f_1} \cdot T_{\text{e2e}}^{\text{l.o. } f_1} + b^{f_1}}{R_{\text{e2e}}^{\text{l.o. } f_1}} \\ &= \frac{15 \cdot 86\frac{1}{9} + 25}{15} \\ &= 87\frac{7}{9} \end{aligned}$$

$$\begin{aligned} B^{f_1} &= \alpha^{f_1}(T_{\text{e2e}}^{\text{l.o. } f_1}) \\ &= 5 \cdot 86\frac{1}{9} + 25 \\ &= 455\frac{5}{9} \end{aligned}$$

References

- [1] Jens B. Schmitt, Frank A. Zdarsky, and Ivan Martinovic. Improving Performance Bounds in Feed-Forward Networks by Paying Multiplexing Only Once. In *14th GI/ITG Conference on Measurement, Modeling, and Evaluation of Computer and Communication Systems (MMB 2008)*, Dortmund, Germany, March 2008. GI/ITG.