

# Network Calculus Tests – Tree (TR) Networks

Version 2.0 beta2 (2017-Jun-25)



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## General Information

- The network calculus analyses presented in this document were created for the purpose of testing the Disco Deterministic Network Calculator (DiscoDNC)<sup>1</sup> – an open-source deterministic network calculus tool developed by the *Distributed Computer Systems (DISCO) Lab* at the University of Kaiserslautern.
- Naming of the individual network configurations depicts the name of the according functional test for the DiscoDNC.
- The naming scheme used in this document is detailed in NetworkCalculus\_NamingScheme.pdf.
- Arrival bound computations are equivalent to the `PbooArrivalBound_Output_PerHop.java` class of the DiscoDNC.
- The end-to-end left-over service curve for PBOO arrival bounds can be computed by simply convolving the server-local ones.
- Arrival bounds for `PmooArrivalBound.java` and analyses using them are listed only if results are different to PBOO.

## Changelog:

Version 1.1 (2014-Dec-30):

- Streamlined the PMOO left-over latency  $T_{e2e}^{l.o.f}$  computation.
- Adapted to naming scheme version 1.1.

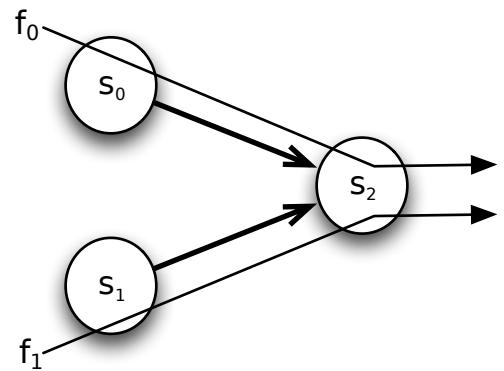
Version 2.0 beta2 (2017-Jun-25):

- Rework of the documentation according to code changes
  - New, more complete naming.
  - Separation of network and test.

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<sup>1</sup><http://disco.cs.uni-kl.de/index.php/projects/disco-dnc>

## TR\_3S\_1SC\_2F\_1AC\_2P\_Network



- $\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0,1,2\}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0,1\}$

## TR\_3S\_1SC\_2F\_1AC\_2P\_Test

arrivalBound( $s_2, \{f_n\}, \mathcal{G}$ ) , $\mathcal{G} = \mathcal{P}(\mathcal{F}) = \alpha_{s_2}^{f_n}, n \in \{0, 1\}$	FIFO_MUX	ARB_MUX
$\alpha_{s_2}^{f_n}$		$= \gamma_{5,25}$
$\alpha_{s_2}^{x f_n}$		$= \gamma_{0,0}$
$\beta_{s_2}^{\text{l.o.} f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x f_n} = \beta_{R_{s_2}^{\text{l.o.} f_n}, T_{s_2}^{\text{l.o.} f_n}}$	$=$	$= \beta_{20,20}$
$\alpha_{s_2}^{f_n} = \alpha_{s_n}^{f_n} \oslash \beta_{s_n}^{\text{l.o.} f_n} = \gamma_{r_{s_2}^{f_n}, b_{s_2}^{f_n}}$	$r_{s_2}^{f_n}$	$= 5$
	$b_{s_2}^{f_n}$	$\alpha_{s_n}^{f_n}(T_{s_n}^{\text{l.o.} f_n}) = 5 \cdot 20 + 25 = 125$
	$=$	$= \gamma_{5,125}$

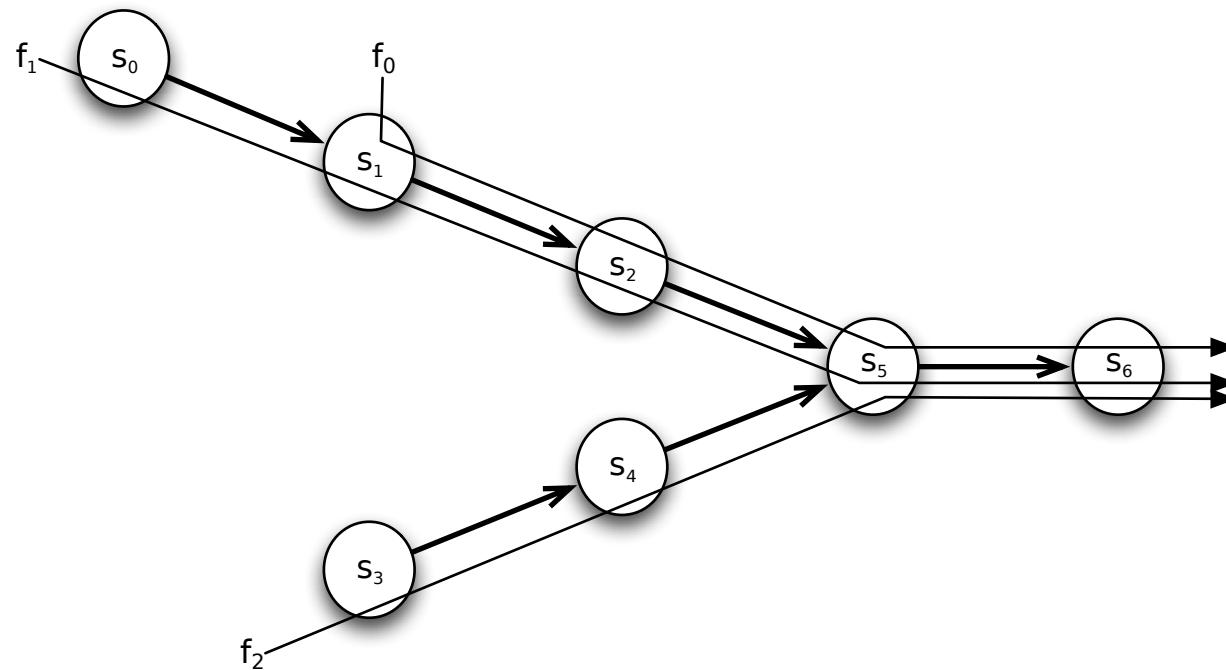
Flow  $f_n$ ,  $n \in \{0, 1\}$  (comparable with Tandem\_1SC\_2Flows\_1AC\_2Paths)

TFA		FIFO_MUX	ARB_MUX
$s_n$	$\alpha_{s_n} = \alpha_{s_n}^{f_n}$	$= \gamma_{5,25}$	
	$D_{s_n}^{f_n}$	$\beta_{s_n} = b_{s_n}^{f_n}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{s_n} = b_{s_n}^{f_n}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_n}^{f_n}$	$\alpha_{s_n}(T_{s_n}) = 5 \cdot 20 + 25$ $= 125$	
$s_2$	$\alpha_{s_2} = \sum_j \alpha_{s_2}^{f_n}$	$= \gamma_{5,125} + \gamma_{5,125} = \gamma_{10,250}$	
	$D_{s_2}^{f_n}$	$\beta_{s_2} = b_{s_1}$ $20 \cdot [t - 20]^+ = 250$ $t = 32\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 250$ $t = 65$
	$B_{s_2}^{f_n}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 250$ $= 450$	
$D^{f_n}$	$\sum_{i=\{n,2\}} D_{s_i}^{f_n} = 53\frac{3}{4}$	$\sum_{i=\{n,2\}} D_{s_i}^{f_n} = 86\frac{1}{4}$	
$B^{f_n}$	$\max_{i=\{n,2\}} b_{s_i}^{f_n} = 450$		

SFA		FIFO_MUX	ARB_MUX
$s_n$	$\alpha_{s_n}^{x f_n}$	$= \gamma_{0,0}$	
	$\beta_{s_n}^{\text{l.o.} f_n} = \beta_{s_n} \ominus \alpha_{s_n}^{x f_n} = \beta_{s_n}$	$= \beta_{20,20}$	
$s_2$	$\alpha_{s_2}^{x f_n} = \alpha_{s_2}^{f_n}$	$= \gamma_{5,125}$	
	$\beta_{s_2}^{\text{l.o.} f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x f_n} = \beta_{R_{s_2}^{\text{l.o.} f_n}, T_{s_2}^{\text{l.o.} f_n}}$	$[R_{s_2} - r_{s_2}^{x f_n}]^+ = 15$	
		$\beta_{s_2} = b_{s_2}^{x f_n}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	$\beta_{s_2} = \alpha_{s_2}^{x f_n}$ $20 \cdot [t - 20]^+ = 5 \cdot t + 125$ $t = 35$
	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$	
$\beta_{e2e}^{\text{l.o.} f_n} = \beta_{R_{e2e}^{\text{l.o.} f_n}, T_{e2e}^{\text{l.o.} f_n}}$	$\bigotimes_{i=\{n,2\}} \beta_{s_i}^{\text{l.o.} f_n} = \beta_{15,46\frac{1}{4}}$	$\bigotimes_{i=\{n,2\}} \beta_{s_i}^{\text{l.o.} f_n} = \beta_{15,55}$	
$D^{f_n}$	$\beta_{e2e}^{\text{l.o.} f_n} = b^{f_n}$ $15 \cdot [t - 46\frac{1}{4}]^+ = 25$ $t = 47\frac{11}{12}$	$\beta_{e2e}^{\text{l.o.} f_n} = b^{f_n}$ $15 \cdot [t - 55]^+ = 25$ $t = 56\frac{2}{3}$	
$B^{f_n}$	$\alpha^{f_n}(T_{e2e}^{\text{l.o.} f_n}) = 5 \cdot 46\frac{1}{4} + 25$ $= 256\frac{1}{4}$	$\alpha^{f_n}(T_{e2e}^{\text{l.o.} f_n}) = 5 \cdot 55 + 25$ $= 300$	

PMOO		ARB_MUX
$s_n$	$\alpha_{s_n}^{\bar{x}f_n}$	$= \gamma_{0,0}$
	$\alpha_{s_n}^{xf_n}$	$= \gamma_{0,0}$
$s_2$	$\alpha_{s_2}^{\bar{x}(f_0)}$	$= \gamma_{5,125}$
	$\alpha_{s_2}^{x(f_0)}$	$= \gamma_{5,125}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$	$R_{e2e}^{l.o.f_n} = \bigwedge_{i \in \{n,2\}} (R_{s_i} - r_{s_i}^{xf_n})$	$= (20 - 0) \wedge (20 - 5)$ $= 15$
	$T_{e2e}^{l.o.f_n} = \sum_{i \in \{n,2\}} \left( T_{s_i} + \frac{b_{s_i}^{\bar{x}f_n} + r_{s_i}^{xf_n} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 20 + \frac{0 + 0 \cdot 20}{15} + 20 + \frac{125 + 5 \cdot 20}{15}$ $= 40 + \frac{225}{15}$ $= 55$
	$=$	$= \beta_{15,55}$
	$D^{f_n}$	$\beta_{e2e}^{l.o.f_n} = b^{f_n}$ $15 \cdot [t - 55]^+ = 25$ $t = 56\frac{2}{3}$
$B^{f_n}$		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 55 + 25$ $= 300$

## TR\_7S\_1SC\_3F\_1AC\_3P\_Network



- $\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0, 1, 2\}$
- $\mathcal{F} = \{f_0, f_1, f_2\}$
- $\alpha^{f_0} = \alpha^{f_1} = \alpha^{f_2} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0, 1, 2\}$

## TR\_7S\_1SC\_3F\_1AC\_3P\_Test

arrivalBound( $s_1, \{f_1\}, \mathcal{G}$ ) , $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_1}^{f_1}$	FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_1}$		$= \gamma_{5,25}$
$\alpha_{s_0}^{x(f_1)}$		$= \gamma_{0,0}$
$\beta_{s_0}^{l.o.f_1} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_1)} = \beta_{R_{s_0}^{l.o.f_1}, T_{s_0}^{l.o.f_1}}$	$=$	$= \beta_{20,20}$
	$r_{s_1}^{f_1}$	$= 5$
$\alpha_{s_1}^{f_1} = \alpha_{s_0}^{f_1} \oslash \beta_{s_0}^{l.o.f_1} = \gamma_{r_{s_1}^{f_1}, b_{s_1}^{f_1}}$	$b_{s_1}^{f_1}$	$\alpha_{s_0}^{f_1}(T_{s_0}^{l.o.f_1}) = 5 \cdot 20 + 25 = 125$
	$=$	$= \gamma_{5,125}$

arrivalBound( $s_2, \{f_1\}, \{f_0\}$ ) = $\alpha_{s_2}^{f_1}$	FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_1}$		$= \gamma_{5,125}$
$\alpha_{s_1}^{x(f_1)}$		$= \gamma_{0,0}$
$\beta_{s_1}^{l.o.f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)} = \beta_{R_{s_1}^{l.o.f_1}, T_{s_1}^{l.o.f_1}}$	$=$	$= \beta_{20,20}$
	$r_{s_2}^{f_1}$	$= 5$
$\alpha_{s_2}^{f_1} = \alpha_{s_1}^{f_1} \oslash \beta_{s_1}^{l.o.f_1} = \gamma_{r_{s_2}^{f_1}, b_{s_2}^{f_1}}$	$b_{s_2}^{f_1}$	$\alpha_{s_1}^{f_1}(T_{s_1}^{l.o.f_1}) = 5 \cdot 20 + 125 = 225$
	$=$	$= \gamma_{5,225}$

arrivalBound( $s_5, \{f_1\}, \{f_0\}$ ) = $\alpha_{s_5}^{f_1}$	FIFO_MUX	ARB_MUX
$\alpha_{s_2}^{f_1}$		$= \gamma_{5,225}$
$\alpha_{s_2}^{x(f_1)}$		$= \gamma_{0,0}$
$\beta_{s_2}^{l.o.f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)} = \beta_{R_{s_2}^{l.o.f_1}, T_{s_2}^{l.o.f_1}}$	$=$	$= \beta_{20,20}$
	$r_{s_5}^{f_1}$	$= 5$
$\alpha_{s_5}^{f_1} = \alpha_{s_2}^{f_1} \oslash \beta_{s_2}^{l.o.f_1} = \gamma_{r_{s_5}^{f_1}, b_{s_5}^{f_1}}$	$b_{s_5}^{f_1}$	$\alpha_{s_2}^{f_1}(T_{s_2}^{l.o.f_1}) = 5 \cdot 20 + 225 = 325$
	$=$	$= \gamma_{5,325}$

arrivalBound( $s_4, \{f_2\}, \mathcal{G}$ ) , $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_4}^{f_2}$	FIFO_MUX	ARB_MUX
$\alpha_{s_3}^{f_2}$		$= \gamma_{5,25}$
$\alpha_{s_3}^{x(f_2)}$		$= \gamma_{0,0}$
$\beta_{s_3}^{l.o.f_2} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_2)} = \beta_{R_{s_3}^{l.o.f_2}, T_{s_3}^{l.o.f_2}}$	$=$	$= \beta_{20,20}$
	$r_{s_4}^{f_2}$	$= 5$
$\alpha_{s_4}^{f_2} = \alpha_{s_3}^{f_2} \oslash \beta_{s_3}^{l.o.f_2} = \gamma_{r_{s_4}^{f_2}, b_{s_4}^{f_2}}$	$b_{s_4}^{f_2}$	$\alpha_{s_3}^{f_2}(T_{s_3}^{l.o.f_2}) = 5 \cdot 20 + 25 = 125$
	$=$	$= \gamma_{5,125}$

arrivalBound( $s_5, \{f_2\}, \mathcal{G}$ ) , $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_5}^{f_2}$	FIFO_MUX	ARB_MUX
$\alpha_{s_4}^{f_2}$		$= \gamma_{5,125}$
$\alpha_{s_4}^{x(f_2)}$		$= \gamma_{0,0}$
$\beta_{s_4}^{l.o.f_2} = \beta_{s_4} \ominus \alpha_{s_4}^{x(f_2)} = \beta_{R_{s_4}^{l.o.f_2}, T_{s_4}^{l.o.f_2}}$	$=$	$= \beta_{20,20}$
	$r_{s_5}^{f_2}$	$= 5$
$\alpha_{s_5}^{f_2} = \alpha_{s_4}^{f_2} \oslash \beta_{s_4}^{l.o.f_2} = \gamma_{r_{s_5}^{f_2}, b_{s_5}^{f_2}}$	$b_{s_5}^{f_2}$	$\alpha_{s_4}^{f_2}(T_{s_4}^{l.o.f_2}) = 5 \cdot 20 + 125 = 225$
	$=$	$= \gamma_{5,225}$

$\text{arrivalBound}(s_6, \{f_1, f_2\}, \{f_0\}) = \alpha_{s_6}^{\{f_1, f_2\}}$	FIFO_MUX	ARB_MUX
$\alpha_{s_5}^{\{f_1, f_2\}}$		$= \gamma_{10,550}$
$\alpha_{s_5}^x\{f_1, f_2\}$		$= \gamma_{0,0}$
$\beta_{s_5}^{\text{l.o.}\{f_1, f_2\}} = \beta_{s_5} \ominus \alpha_{s_5}^x\{f_1, f_2\} = \beta_{R_{s_5}^{\text{l.o.}\{f_1, f_2\}}, T_{s_5}^{\text{l.o.}\{f_1, f_2\}}}$	$=$	$= \beta_{20,20}$
$\alpha_{s_6}^{\{f_1, f_2\}} = \alpha_{s_5}^{\{f_1, f_2\}} \oslash \beta_{s_5}^{\text{l.o.}\{f_1, f_2\}} = \gamma_{r_{s_6}^{\{f_1, f_2\}}, b_{s_6}^{\{f_1, f_2\}}}$	$r_{s_6}^{\{f_1, f_2\}}$	$= 10$
	$b_{s_6}^{\{f_1, f_2\}}$	$\alpha_{s_5}^{\{f_1, f_2\}}(T_{s_5}^{\text{l.o.}\{f_1, f_2\}}) = 10 \cdot 20 + 550 = 750$
	$=$	$= \gamma_{10,750}$

$\text{arrivalBound}(s_2, \{f_0, f_1\}, \mathcal{G}), \mathcal{G} \in \mathcal{P}(\{f_2\}) = \alpha_{s_2}^{\{f_0, f_1\}}$	FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{\{f_0, f_1\}}$		$= \gamma_{10,150}$
$\alpha_{s_1}^x\{f_0, f_1\}$		$= \gamma_{0,0}$
$\beta_{s_1}^{\text{l.o.}\{f_0, f_1\}} = \beta_{s_1} \ominus \alpha_{s_1}^x\{f_0, f_1\} = \beta_{R_{s_1}^{\text{l.o.}\{f_0, f_1\}}, T_{s_1}^{\text{l.o.}\{f_0, f_1\}}}$	$=$	$= \beta_{20,20}$
$\alpha_{s_2}^{\{f_0, f_1\}} = \alpha_{s_1}^{\{f_0, f_1\}} \oslash \beta_{s_1}^{\text{l.o.}\{f_0, f_1\}} = \gamma_{r_{s_2}^{\{f_0, f_1\}}, b_{s_2}^{\{f_0, f_1\}}}$	$r_{s_2}^{\{f_0, f_1\}}$	$= 10$
	$b_{s_2}^{\{f_0, f_1\}}$	$\alpha_{s_1}^{\{f_0, f_1\}}(T_{s_1}^{\text{l.o.}\{f_0, f_1\}}) = 10 \cdot 20 + 150 = 350$
	$=$	$= \gamma_{10,350}$

$\text{arrivalBound}(s_5, \{f_0, f_1\}, \mathcal{G}), \mathcal{G} \in \mathcal{P}(\{f_2\}) = \alpha_{s_5}^{\{f_0, f_1\}}$	FIFO_MUX	ARB_MUX
$\alpha_{s_2}^{\{f_0, f_1\}}$		$= \gamma_{10,350}$
$\alpha_{s_2}^x\{f_0, f_1\}$		$= \gamma_{0,0}$
$\beta_{s_2}^{\text{l.o.}\{f_0, f_1\}} = \beta_{s_2} \ominus \alpha_{s_2}^x\{f_0, f_1\} = \beta_{R_{s_2}^{\text{l.o.}\{f_0, f_1\}}, T_{s_2}^{\text{l.o.}\{f_0, f_1\}}}$	$=$	$= \beta_{20,20}$
$\alpha_{s_5}^{\{f_0, f_1\}} = \alpha_{s_2}^{\{f_0, f_1\}} \oslash \beta_{s_2}^{\text{l.o.}\{f_0, f_1\}} = \gamma_{r_{s_5}^{\{f_0, f_1\}}, b_{s_5}^{\{f_0, f_1\}}}$	$r_{s_5}^{\{f_0, f_1\}}$	$= 10$
	$b_{s_5}^{\{f_0, f_1\}}$	$\alpha_{s_2}^{\{f_0, f_1\}}(T_{s_2}^{\text{l.o.}\{f_0, f_1\}}) = 10 \cdot 20 + 350 = 550$
	$=$	$= \gamma_{10,550}$

$\text{arrivalBound}(s_6, \{f_0, f_1, f_2\}, \{\}) = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$	FIFO_MUX	ARB_MUX
$\alpha_{s_5}^{\{f_0, f_1, f_2\}}$		$= \gamma_{10,775}$
$\alpha_{s_5}^x\{f_0, f_1, f_2\}$		$= \gamma_{0,0}$
$\beta_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}} = \beta_{s_5} \ominus \alpha_{s_5}^x\{f_0, f_1, f_2\} = \beta_{R_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}}, T_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}}}$	$=$	$= \beta_{20,20}$
$\alpha_{s_6}^{\{f_0, f_1, f_2\}} = \alpha_{s_5}^{\{f_0, f_1, f_2\}} \oslash \beta_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}} = \gamma_{r_{s_6}^{\{f_0, f_1, f_2\}}, b_{s_6}^{\{f_0, f_1, f_2\}}}$	$r_{s_6}^{\{f_0, f_1, f_2\}}$	$= 15$
	$b_{s_6}^{\{f_0, f_1, f_2\}}$	$\alpha_{s_5}^{\{f_0, f_1, f_2\}}(T_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}}) = 15 \cdot 20 + 775 = 1075$
	$=$	$= \gamma_{15,1075}$

**Flow  $f_0$**

TFA		FIFO_MUX	ARB_MUX
$s_1$	$\alpha_{s_1} = \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_1}$	$= \gamma_{5,25} + \gamma_{5,125} = \gamma_{10,150}$	
	$D_{s_1}^{f_0}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 150$ $t = 27\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 150$ $t = 55$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 150$ = 350	
$s_2$	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$	$= \gamma_{10,350}$	
	$D_{s_2}^{f_0}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t - 20]^+ = 350$ $t = 37\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 350$ $t = 75$
	$B_{s_2}^{f_0}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 350$ = 550	
$s_5$	$\alpha_{s_5} = \alpha_{s_5}^{\{f_0, f_1\}} + \alpha_{s_5}^{f_2}$	$= \gamma_{10,550} + \gamma_{5,225} = \gamma_{15,775}$	
	$D_{s_5}^{f_0}$	$\beta_{s_5} = b_{s_5}$ $20 \cdot [t - 20]^+ = 775$ $t = 58\frac{3}{4}$	$\beta_{s_5} = \alpha_{s_5}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 775$ $t = 235$
	$B_{s_5}^{f_0}$	$\alpha_{s_5}(T_{s_5}) = 15 \cdot 20 + 775$ = 1075	
$s_6$	$\alpha_{s_6} = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$	$= \gamma_{15,1075}$	
	$D_{s_6}^{f_0}$	$\beta_{s_6} = b_{s_6}$ $20 \cdot [t - 20]^+ = 1075$ $t = 73\frac{3}{4}$	$\beta_{s_6} = \alpha_{s_6}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 1075$ $t = 295$
	$B_{s_6}^{f_0}$	$\alpha_{s_6}(T_{s_6}) = 15 \cdot 20 + 1075$ = 1375	
$D^{f_0}$		$\sum_{i=\{1,2,5,6\}} D_{s_i}^{f_0} = 27\frac{1}{2} + 37\frac{1}{2} + 58\frac{3}{4} + 73\frac{3}{4} = 197\frac{1}{2}$	$\sum_{i=\{1,2,5,6\}} D_{s_i}^{f_0} = 55 + 75 + 235 + 295 = 660$
$B^{f_0}$		$\max_{i=\{1,2,5,6\}} B_{s_i}^{f_0} = 1375$	

SFA FIFO\_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f_0} &= \left( \beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left( \beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left( \beta_{s_5}^{l.o.x(f_0)} \ominus \alpha_{s_5}^{x(f_0)} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{x(f_0)} \right) \\
&= \left( \beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left( \beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left( \beta_{s_5}^{l.o.x(f_0)} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left( \beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{\{f_1, f_2\}} \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( \alpha_{s_5}^{\{f_1, f_2\}} \oslash \beta_{s_5}^{\{f_1, f_2\}} \right) \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash \beta_{s_5}^{\{f_1, f_2\}} \right) \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash (\beta_{s_5} \ominus \alpha_{s_5}^{\{f_1, f_2\}}) \right) \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash \beta_{s_5} \right) \right) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus (\gamma_{5,325} + \gamma_{5,225})) \otimes (\beta_{20,20} \ominus ((\gamma_{5,325} + \gamma_{5,225}) \oslash \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus (\gamma_{10,550} \oslash \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus \gamma_{10,750}) \\
&= \beta_{15,26\frac{1}{4}} \otimes \beta_{15,31\frac{1}{4}} \otimes \beta_{10,47\frac{1}{2}} \otimes \beta_{10,57\frac{1}{2}} \\
&= \beta_{10,162\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
D^{f_0} &= \frac{R_{e2e}^{l.o.f_0} \cdot T_{e2e}^{l.o.f_0} + b^{f_0}}{R_{e2e}^{l.o.f_0}} \\
&= \frac{10 \cdot 162\frac{1}{2} + 25}{10} \\
&= 165
\end{aligned}$$

$$\begin{aligned}
B^{f_0} &= \alpha^{f_0}(T_{e2e}^{l.o.f_0}) \\
&= 5 \cdot 162\frac{1}{2} + 25 \\
&= 837\frac{1}{2}
\end{aligned}$$

SFA ARB\_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f_0} &= \left( \beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left( \beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left( \beta_{s_5}^{l.o.x(f_0)} \ominus \alpha_{s_5}^{x(f_0)} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{x(f_0)} \right) \\
&= \left( \beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left( \beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left( \beta_{s_5}^{l.o.x(f_0)} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left( \beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{\{f_1, f_2\}} \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( \alpha_{s_5}^{\{f_1, f_2\}} \oslash \beta_{s_5}^{\{f_1, f_2\}} \right) \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash \beta_{s_5}^{\{f_1, f_2\}} \right) \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash (\beta_{s_5} \ominus \alpha_{s_5}^{\{f_1, f_2\}}) \right) \right) \\
&= (\beta_{s_1} \ominus \alpha_{s_1}^{f_1}) \otimes (\beta_{s_2} \ominus \alpha_{s_2}^{f_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2})) \otimes \left( \beta_{s_6} \ominus \left( (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash \beta_{s_5} \right) \right) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus (\gamma_{5,325} + \gamma_{5,225})) \otimes (\beta_{20,20} \ominus ((\gamma_{5,325} + \gamma_{5,225}) \oslash \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus (\gamma_{10,550} \oslash \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus \gamma_{10,750}) \\
&= \beta_{15,35} \otimes \beta_{15,41\frac{2}{3}} \otimes \beta_{10,95} \otimes \beta_{10,115} \\
&= \beta_{10,286\frac{2}{3}}
\end{aligned}$$

$$\begin{aligned}
D^{f_0} &= \frac{R_{e2e}^{l.o.f_0} \cdot T_{e2e}^{l.o.f_0} + b^{f_0}}{R_{e2e}^{l.o.f_0}} \\
&= \frac{10 \cdot 286\frac{2}{3} + 25}{10} \\
&= 289\frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
B^{f_0} &= \alpha^{f_0}(T_{e2e}^{l.o.f_0}) \\
&= 5 \cdot 286\frac{2}{3} + 25 \\
&= 1458\frac{1}{3}
\end{aligned}$$

PMOO		ARB_MUX
$s_1$	$\alpha_{s_1}^{\bar{x}(f_0)}$	$= \gamma_{5,125}$
	$\alpha_{s_1}^{x(f_0)}$	$= \gamma_{5,125}$
$s_2$	$\alpha_{s_2}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_2}^{x(f_0)}$	$= \gamma_{5,125}$
$s_5$	$\alpha_{s_5}^{\bar{x}(f_0)}$	$= \gamma_{5,225}$
	$\alpha_{s_5}^{x(f_0)}$	$= \gamma_{10,xxx}$
$s_6$	$\alpha_{s_6}^{\bar{x}(f_0)}$	$= \gamma_{0,0}$
	$\alpha_{s_6}^{x(f_0)}$	$= \gamma_{10,xxx}$
$\beta_{e2e}^{l.o.f_0} = \beta_{R_{e2e}^{l.o.f_0}, T_{e2e}^{l.o.f_0}}$	$R_{e2e}^{l.o.f_0} = \bigwedge_{i \in \{1,2,5,6\}} (R_{s_i} - r_{s_i}^{x(f_0)})$	$= (20 - 5) \wedge (20 - 5) \wedge (20 - 10) \wedge (20 - 10)$ $= \frac{10}{10}$
	$T_{e2e}^{l.o.f_0} = \sum_{i \in \{1,2,5,6\}} \left( T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 20 + \frac{125 + 5 \cdot 20}{10} + 20 + \frac{0 + 5 \cdot 20}{10} + 20 + \frac{225 + 10 \cdot 20}{10} + 20 + \frac{0 + 10 \cdot 20}{10}$ $= 80 + \frac{950}{10}$ $= \frac{175}{175}$
	$=$	$= \beta_{10,185}$
	$D^{f_0}$	$\beta_{e2e}^{l.o.f_0} = b^{f_0}$ $10 \cdot [t - 175]^+ = 25$ $t = 177 \frac{1}{2}$
$B^{f_0}$		$\alpha^{f_0}(T_{e2e}^{l.o.f_0}) = 5 \cdot 175 + 25$ $= 900$

**Flow  $f_1$**

TFA		FIFO_MUX	ARB_MUX
$s_0$	$\alpha_{s_1} = \alpha_{s_1}^{f_1}$		$= \gamma_{5,25}$
	$D_{s_0}^{f_1}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_0}^{f_1}$	$\alpha_{s_0}(T_{s_0}) = 5 \cdot 20 + 25$ = 125	
$s_1$	$\alpha_{s_1} = \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_1}$		$= \gamma_{5,25} + \gamma_{5,125} = \gamma_{10,150}$
	$D_{s_1}^{f_1}$	$= 27\frac{1}{2}$	= 55
	$B_{s_1}^{f_1}$		= 350
$s_2$	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$		$= \gamma_{10,350}$
	$D_{s_2}^{f_1}$	$= 37\frac{1}{2}$	= 75
	$B_{s_2}^{f_1}$		= 550
$s_5$	$\alpha_{s_5} = \alpha_{s_5}^{\{f_0, f_1\}} + \alpha_{s_5}^{f_2}$		$= \gamma_{5,225} + \gamma_{10,550} = \gamma_{15,775}$
	$D_{s_5}^{f_1}$	$= 58\frac{3}{4}$	= 235
	$B_{s_5}^{f_1}$		= 1075
$s_6$	$\alpha_{s_6} = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$		$= \gamma_{15,1075}$
	$D_{s_6}^{f_1}$	$= 73\frac{3}{4}$	= 295
	$B_{s_6}^{f_1}$		= 1375
$D^{f_1}$		$\sum_{i \in \{0,1,2,5,6\}} D_{s_i}^{f_1} = 21\frac{1}{4} + 27\frac{1}{2} + 37\frac{1}{2} + 58\frac{3}{4} + 73\frac{3}{4} = 218\frac{3}{4}$	$\sum_{i \in \{0,1,2,5,6\}} D_{s_i}^{f_1} = 21\frac{1}{4} + 55 + 75 + 235 + 295 = 681\frac{1}{4}$
$B^{f_1}$		$\max_{i \in \{0,1,2,5,6\}} B_{s_i}^{f_1} = 1375$	

### SFA FIFO MUX:

$$D^{f_1} = \frac{R_{\text{e2e}}^{\text{l.o.} f_1} \cdot T_{\text{e2e}}^{\text{l.o.} f_1} + b^{f_1}}{R_{\text{e2e}}^{\text{l.o.} f_1}}$$

$$= \frac{10 \cdot 162\frac{1}{2} + 25}{10}$$

= 165

$$B^{f_1} = \alpha^{f_1} \left( T_{\text{e2e}}^{\text{l.o.} f_1} \right)$$

$$= 5 \cdot 162 \frac{1}{2} + 25$$

$$= 837\frac{1}{2}$$

SFA ARB\_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f_1} &= \left( \beta_{s_0}^{l.o.x(f_1)} \ominus \alpha_{s_0}^{x(f_1)} \right) \otimes \left( \beta_{s_1}^{l.o.x(f_1)} \ominus \alpha_{s_1}^{x(f_1)} \right) \otimes \left( \beta_{s_2}^{l.o.x(f_1)} \ominus \alpha_{s_2}^{x(f_1)} \right) \otimes \left( \beta_{s_5}^{l.o.x(f_1)} \ominus \alpha_{s_5}^{x(f_1)} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_1)} \ominus \alpha_{s_6}^{x(f_1)} \right) \\
&= \left( \beta_{s_0}^{l.o.x(f_1)} \ominus \alpha_{s_0}^{x(f_1)} \right) \otimes \left( \beta_{s_1}^{l.o.x(f_1)} \ominus \alpha_{s_1}^{x(f_1)} \right) \otimes \left( \beta_{s_2}^{l.o.x(f_1)} \ominus \left( \alpha_{s_1}^{x(f_1)} \otimes \beta_{s_1}^{l.o.x(f_1)} \right) \right) \otimes \left( \beta_{s_5}^{l.o.x(f_1)} - \alpha_{s_5}^{x(f_1)} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_1)} - \left( \alpha_{s_5}^{x(f_1)} \otimes \beta_{s_5}^{l.o.x(f_1)} \right) \right) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1})) \otimes (\beta_{s_5} \ominus \alpha_{s_5}^{x(f_1)}) \otimes (\beta_{s_6} \ominus (\alpha_{s_5}^{x(f_1)} \otimes \beta_{s_5}^{l.o.x(f_1)})) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus ((\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_0} + \alpha_{s_5}^{f_2}))) \otimes (\beta_{s_6} \ominus (((\alpha_{s_5}^{f_0} + \alpha_{s_5}^{f_2}) \otimes \beta_{s_5}^{l.o.\{f_0,f_2\}}))) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_0} + \alpha_{s_5}^{f_2})) \otimes (\beta_{s_6} \ominus (((\alpha_{s_5}^{f_0} + \alpha_{s_5}^{f_2}) \otimes \beta_{s_5}))) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_0} + \alpha_{s_5}^{f_2})) \otimes (\beta_{s_6} \ominus (((\alpha_{s_5}^{f_0} + \alpha_{s_5}^{f_2}) \otimes \beta_{s_5}))) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_0} + (\alpha_{s_4}^{f_2} \otimes \beta_{s_4}))) \otimes (\beta_{s_6} \ominus (((\alpha_{s_5}^{f_0} + (\alpha_{s_4}^{f_2} \otimes \beta_{s_4})) \otimes \beta_{s_5}))) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus (\alpha_{s_5}^{f_0} + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4}))) \otimes (\beta_{s_6} \ominus (((\alpha_{s_5}^{f_0} + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4})) \otimes \beta_{s_5}))) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus ((\alpha_{s_2}^{f_0} \otimes \beta_{s_2}) + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4}))) \otimes (\beta_{s_6} \ominus ((((\alpha_{s_2}^{f_0} \otimes \beta_{s_2}) + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4})) \otimes \beta_{s_5}))) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus (((\alpha_{s_1}^{f_0} \otimes \beta_{s_1}^{l.o.f_0}) \otimes \beta_{s_2}) + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4}))) \otimes (\beta_{s_6} \ominus (((((\alpha_{s_1}^{f_0} \otimes \beta_{s_1}^{l.o.f_0}) \otimes \beta_{s_2}) + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4})) \otimes \beta_{s_5}))) \\
&= \beta_{s_0} \otimes (\beta_{s_1} \ominus \alpha^{f_0}) \otimes (\beta_{s_2} \ominus (\alpha^{f_0} \otimes \beta_{s_1}) \otimes (\beta_{s_5} \ominus ((((\alpha_{s_1}^{f_0} \otimes \beta_{s_1}) \otimes \beta_{s_2}) + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4}))) \otimes (\beta_{s_6} \ominus (((((\alpha_{s_1}^{f_0} \otimes \beta_{s_1}) \otimes \beta_{s_2}) + ((\alpha^{f_2} \otimes \beta_{s_3}) \otimes \beta_{s_4})) \otimes \beta_{s_5}))) \\
&= \beta_{20,20} \otimes (\beta_{20,20} \ominus \gamma_{5,25}) \otimes (\beta_{20,20} \ominus (\gamma_{5,25} \otimes \beta_{20,20})) \\
&\quad \otimes (\beta_{20,20} \ominus (((\gamma_{5,25} \otimes \beta_{20,20}) \otimes \beta_{20,20}) + ((\gamma_{5,25} \otimes \beta_{20,20}) \otimes \beta_{20,20}))) \otimes (\beta_{20,20} \ominus (((((\gamma_{5,25} \otimes \beta_{20,20}) \otimes \beta_{20,20}) + ((\gamma_{5,25} \otimes \beta_{20,20}) \otimes \beta_{20,20})) \otimes \beta_{20,20})) \\
&= \beta_{20,20} \otimes \beta_{20,28\frac{1}{3}} \otimes (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus ((\gamma_{5,125} \otimes \beta_{20,20}) + (\gamma_{5,125} \otimes \beta_{20,20}))) \otimes (\beta_{20,20} \ominus (((\gamma_{5,125} \otimes \beta_{20,20}) + (\gamma_{5,125} \otimes \beta_{20,20})) \otimes \beta_{20,20})) \\
&= \beta_{20,20} \otimes \beta_{15,28\frac{1}{3}} \otimes \beta_{15,35} \otimes (\beta_{20,20} \ominus (\gamma_{5,225} + \gamma_{5,225})) \otimes (\beta_{20,20} \ominus ((\gamma_{5,225} + \gamma_{5,225}) \otimes \beta_{20,20})) \\
&= \beta_{20,20} \otimes \beta_{15,28\frac{1}{3}} \otimes \beta_{15,35} \otimes (\beta_{20,20} \ominus \gamma_{10,450}) \otimes (\beta_{20,20} \ominus (\gamma_{10,450} \otimes \beta_{20,20})) \\
&= \beta_{20,20} \otimes \beta_{15,28\frac{1}{3}} \otimes \beta_{15,35} \otimes \beta_{10,85} \otimes (\beta_{20,20} \ominus \gamma_{10,650}) \\
&= \beta_{20,20} \otimes \beta_{15,28\frac{1}{3}} \otimes \beta_{15,35} \otimes \beta_{10,85} \otimes \beta_{10,95} \\
&= \beta_{10,273\frac{1}{3}}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{e2e}^{l.o.f_1} \cdot T_{e2e}^{l.o.f_1} + b^{f_1}}{R_{e2e}^{l.o.f_1}} \\
&= \frac{10 \cdot 273\frac{1}{3} + 25}{10} \\
&= 275\frac{5}{6}
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1} \left( T_{e2e}^{l.o.f_1} \right) \\
&= 5 \cdot 273\frac{1}{3} + 25 \\
&= 1391\frac{2}{3}
\end{aligned}$$

PMOO		ARB_MUX
$s_0$	$\alpha_{s_0}^{\bar{x}(f_1)}$	$= \gamma_{0,0}$
	$\alpha_{s_0}^x(f_1)$	$= \gamma_{0,0}$
$s_1$	$\alpha_{s_1}^{\bar{x}(f_1)}$	$= \gamma_{5,25}$
	$\alpha_{s_1}^x(f_1)$	$= \gamma_{5,25}$
$s_2$	$\alpha_{s_2}^{\bar{x}(f_1)}$	$= \gamma_{0,0}$
	$\alpha_{s_2}^x(f_1)$	$= \gamma_{5,125}$
$s_5$	$\alpha_{s_5}^{\bar{x}(f_1)}$	$= \gamma_{5,225}$
	$\alpha_{s_5}^x(f_1)$	$= \gamma_{10,xxx}$
$s_6$	$\alpha_{s_6}^{\bar{x}(f_1)}$	$= \gamma_{0,0}$
	$\alpha_{s_6}^x(f_1)$	$= \gamma_{10,xxx}$
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$	$R_{e2e}^{l.o.f_1} = \bigwedge_{i \in \{0,1,2,5,6\}} (R_{s_i} - r_{s_i}^x(f_1))$ $T_{e2e}^{l.o.f_1} = \sum_{i \in \{0,1,2,5,6\}} \left( T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_1)} + r_{s_i}^x(f_1) \cdot T_{s_i}}{R_{e2e}^{l.o.f_1}} \right)$ $=$	$= (20 - 0) \wedge (20 - 5) \wedge (20 - 5) \wedge (20 - 10) \wedge (20 - 10)$ $= \frac{10}{10}$ $= 20 + \frac{0 + 0 \cdot 20}{10} + 20 + \frac{25 + 5 \cdot 20}{10} + 20 + \frac{0 + 5 \cdot 20}{10} + 20 + \frac{225 + 10 \cdot 20}{10} + 20 + \frac{0 + 10 \cdot 20}{10}$ $= 100 + \frac{850}{10}$ $= 185$ $= \beta_{10,185}$
$D^{f_1}$		$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $10 \cdot [t - 185]^+ = 25$ $t = 187 \frac{1}{2}$
$B^{f_1}$		$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 185 + 25$ $= 950$

**Flow  $f_2$**

TFA		FIFO_MUX	ARB_MUX
$s_3$	$\alpha_{s_3} = \alpha_{s_3}^{f_2}$		$= \gamma_{5,25}$
	$D_{s_3}^{f_2}$	$\beta_{s_3} = b_{s_3}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{s_3} = b_{s_3}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_3}^{f_2}$	$\alpha_{s_3}(T_{s_3}) = 5 \cdot 20 + 25$ = 125	
$s_4$	$\alpha_{s_4} = \alpha_{s_4}^{f_2}$		$= \gamma_{5,125}$
	$D_{s_4}^{f_2}$	$\beta_{s_4} = b_{s_4}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	FIFO per micro flow $\beta_{s_4} = b_{s_4}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$
	$B_{s_4}^{f_2}$	$\alpha_{s_4}(T_{s_4}) = 5 \cdot 20 + 125$ = 225	
$s_5$	$\alpha_{s_5} = \alpha_{s_5}^{\{f_0, f_1\}} + \alpha_{s_5}^{f_2}$		$= \gamma_{5,225} + \gamma_{10,550} = \gamma_{15,775}$
	$D_{s_5}^{f_2}$	$= 58\frac{3}{4}$	$= 235$
	$B_{s_5}^{f_2}$		$= 1075$
$s_6$	$\alpha_{s_6} = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$		$= \gamma_{15,1075}$
	$D_{s_6}^{f_2}$	$= 73\frac{3}{4}$	$= 295$
	$B_{s_6}^{f_2}$		$= 1375$
$D^{f_2}$		$\sum_{i \in \{3,4,5,6\}} D_{s_i}^{f_2} = 180$	$\sum_{i \in \{3,4,5,6\}} D_{s_i}^{f_2} = 577\frac{1}{2}$
$B^{f_2}$		$\max_{i \in \{3,4,5,6\}} B_{s_i}^{f_2} = 1375$	

SFA FIFO\_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f_2} &= \left( \beta_{s_3}^{l.o.x(f_2)} \ominus \alpha_{s_3}^{x(f_2)} \right) \otimes \left( \beta_{s_4}^{l.o.x(f_2)} \ominus \alpha_{s_4}^{x(f_2)} \right) \otimes \left( \beta_{s_5}^{l.o.x(f_2)} \ominus \alpha_{s_5}^{x(f_2)} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_2)} \ominus \alpha_{s_6}^{x(f_2)} \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \left( \beta_{s_5} \ominus \alpha_{s_5}^{xx(f_2)} \right) \ominus \alpha_{s_5}^{\{f_0,f_1\}} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_2)} \ominus \left( \alpha_{s_5}^{x(f_2)} \oslash \beta_{s_5}^{l.o.x(f_2)} \right) \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \beta_{s_5} \ominus \alpha_{s_5}^{\{f_0,f_1\}} \right) \otimes \left( \beta_{s_6} \ominus \left( \alpha_{s_5}^{\{f_0,f_1\}} \oslash \beta_{s_5} \right) \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \beta_{s_5} \ominus \left( \alpha_{s_1}^{\{f_0,f_1\}} \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \right) \otimes \left( \beta_{s_6} \ominus \left( \left( \alpha_{s_1}^{\{f_0,f_1\}} \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \oslash \beta_{s_5} \right) \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \beta_{s_5} \ominus \left( \left( (\alpha^{f_1} \oslash \beta_{s_0}) + \alpha^{f_0} \right) \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \right) \otimes \left( \beta_{s_6} \ominus \left( \left( \left( (\alpha^{f_1} \oslash \beta_{s_0}) + \alpha^{f_0} \right) \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \oslash \beta_{s_5} \right) \right) \\
&= \beta_{20,20} \otimes \beta_{20,20} \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash (\beta_{20,20} \otimes \beta_{20,20}) \right) \right) \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash (\beta_{20,20} \otimes \beta_{20,20}) \right) \otimes \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash \beta_{20,40} \right) \right) \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash \beta_{20,40} \right) \otimes \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \left( (\gamma_{5,125} + \gamma_{5,25}) \oslash \beta_{20,40} \right) \right) \otimes \left( \beta_{20,20} \ominus \left( (\gamma_{5,125} + \gamma_{5,25}) \oslash \beta_{20,40} \right) \otimes \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus (\gamma_{10,150} \oslash \beta_{20,40}) \right) \otimes \left( \beta_{20,20} \ominus ((\gamma_{10,150} \oslash \beta_{20,40}) \otimes \beta_{20,20}) \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left( \beta_{20,20} \ominus (\gamma_{10,550} \oslash \beta_{20,20}) \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left( \beta_{20,20} \ominus \gamma_{10,750} \right) \\
&= \beta_{20,40} \otimes \beta_{10,47\frac{1}{2}} \otimes \beta_{10,55\frac{1}{2}} \\
&= \beta_{10,145}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{e2e}^{l.o.f_1} \cdot T_{e2e}^{l.o.f_1} + b^{f_1}}{R_{e2e}^{l.o.f_1}} \\
&= \frac{10 \cdot 145 + 25}{10} \\
&= 147\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1}(T_{e2e}^{l.o.f_1}) \\
&= 5 \cdot 145 + 25 \\
&= 750
\end{aligned}$$

SFA ARB\_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f_2} &= \left( \beta_{s_3}^{l.o.x(f_2)} \ominus \alpha_{s_3}^{x(f_2)} \right) \otimes \left( \beta_{s_4}^{l.o.x(f_2)} \ominus \alpha_{s_4}^{x(f_2)} \right) \otimes \left( \beta_{s_5}^{l.o.x(f_2)} \ominus \alpha_{s_5}^{x(f_2)} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_2)} \ominus \alpha_{s_6}^{x(f_2)} \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \left( \beta_{s_5} \ominus \alpha_{s_5}^{xx(f_2)} \right) \ominus \alpha_{s_5}^{\{f_0,f_1\}} \right) \otimes \left( \beta_{s_6}^{l.o.x(f_2)} \ominus \left( \alpha_{s_5}^{x(f_2)} \oslash \beta_{s_5}^{l.o.x(f_2)} \right) \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \beta_{s_5} \ominus \alpha_{s_5}^{\{f_0,f_1\}} \right) \otimes \left( \beta_{s_6} \ominus \left( \alpha_{s_5}^{\{f_0,f_1\}} \oslash \beta_{s_5} \right) \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \beta_{s_5} \ominus \left( \alpha_{s_1}^{\{f_0,f_1\}} \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \right) \otimes \left( \beta_{s_6} \ominus \left( \left( \alpha_{s_1}^{\{f_0,f_1\}} \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \oslash \beta_{s_5} \right) \right) \\
&= \beta_{s_3} \otimes \beta_{s_4} \otimes \left( \beta_{s_5} \ominus \left( \left( (\alpha^{f_1} \oslash \beta_{s_0}) + \alpha^{f_0} \right) \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \right) \otimes \left( \beta_{s_6} \ominus \left( \left( \left( (\alpha^{f_1} \oslash \beta_{s_0}) + \alpha^{f_0} \right) \oslash (\beta_{s_1} \otimes \beta_{s_2}) \right) \oslash \beta_{s_5} \right) \right) \\
&= \beta_{20,20} \otimes \beta_{20,20} \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash (\beta_{20,20} \otimes \beta_{20,20}) \right) \right) \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash (\beta_{20,20} \otimes \beta_{20,20}) \right) \oslash \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash \beta_{20,40} \right) \right) \otimes \left( \beta_{20,20} \ominus \left( \left( (\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash \beta_{20,40} \right) \oslash \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \left( (\gamma_{5,125} + \gamma_{5,25}) \oslash \beta_{20,40} \right) \right) \otimes [\beta_{20,20} - \left( (\gamma_{5,125} + \gamma_{5,25}) \oslash \beta_{20,40} \right) \oslash \beta_{20,20}]^+ \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus (\gamma_{10,150} \oslash \beta_{20,40}) \right) \otimes \left( \beta_{20,20} \ominus ((\gamma_{10,150} \oslash \beta_{20,40}) \oslash \beta_{20,20}) \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left( \beta_{20,20} \ominus (\gamma_{10,550} \oslash \beta_{20,20}) \right) \\
&= \beta_{20,40} \otimes \left( \beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left( \beta_{20,20} \ominus \gamma_{10,750} \right) \\
&= \beta_{20,40} \otimes \beta_{10,95} \otimes \beta_{10,115} \\
&= \beta_{10,250}
\end{aligned}$$

$$\begin{aligned}
D^{f_1} &= \frac{R_{e2e}^{l.o.f_1} \cdot T_{e2e}^{l.o.f_1} + b^{f_1}}{R_{e2e}^{l.o.f_1}} \\
&= \frac{10 \cdot 250 + 25}{10} \\
&= 252\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
B^{f_1} &= \alpha^{f_1}(T_{e2e}^{l.o.f_1}) \\
&= 5 \cdot 250 + 25 \\
&= 1275
\end{aligned}$$

PMOO		ARB_MUX	
$s_3$	$\alpha_{s_3}^{\bar{x}(f_2)}$		$= \gamma_{0,0}$
	$\alpha_{s_3}^{x(f_2)}$		$= \gamma_{0,0}$
$s_4$	$\alpha_{s_4}^{\bar{x}(f_2)}$		$= \gamma_{0,0}$
	$\alpha_{s_4}^{x(f_2)}$		$= \gamma_{0,0}$
$s_5$	$\alpha_{s_5}^{\bar{x}(f_2)}$		$= \gamma_{10,550}$
	$\alpha_{s_5}^{x(f_2)}$		$= \gamma_{10,550}$
$s_6$	$\alpha_{s_6}^{\bar{x}(f_2)}$		$= \gamma_{0,0}$
	$\alpha_{s_6}^{x(f_2)}$		$= \gamma_{10,xxx}$
$\beta_{e2e}^{l.o.f_2} = \beta_{R_{e2e}^{l.o.f_2}, T_{e2e}^{l.o.f_2}}$	$R_{e2e}^{l.o.f_2} = \bigwedge_{i \in \{3,4,5,6\}} (R_{s_i} - r_{s_i}^{x(f_2)})$	$= (20 - 0) \wedge (20 - 0) \wedge (20 - 10) \wedge (20 - 10)$ $= \frac{10}{10}$	
	$T_{e2e}^{l.o.f_2} = \sum_{i \in \{3,4,5,6\}} \left( T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_2)} + r_{s_i}^{x(f_2)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_2}} \right)$	$= 20 + \frac{0 + 0 \cdot 20}{5} + 20 + \frac{0 + 0 \cdot 20}{5} + 20 + \frac{550 + 10 \cdot 20}{5} + 20 + \frac{0 + 10 \cdot 20}{5}$ $= \frac{80 + \frac{950}{10}}{175}$	
	$=$	$= \beta_{10,175}$	
	$D^{f_2}$	$\beta_{e2e}^{l.o.f_2} = b^{f_2}$ $10 \cdot [t - 175]^+ = 25$ $t = 177\frac{1}{2}$	
	$B^{f_2}$	$\alpha^{f_2}(T_{e2e}^{l.o.f_2}) = 5 \cdot 175 + 25$ $= 900$	